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Break of Logic Symmetry by Self-conflicting Agents: Descriptive vs. Prescriptive Rules

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Abstract — *Classical axiomatic uncertainty theories (probability theory and others) model reasoning of rational agents. These theories are prescriptive, i.e., prescribe how a rational agent should reason about uncertainties. In particular, it is prescribed that (1) uncertainty P of any sentence p is evaluated by a single scalar P(p) value, (2) the truth-value of any tautology ($p \vee \neg p$) is true, and (3) the truth-value of any contradiction ($p \wedge \neg p$) is false for every proposition p. However, real agents can be quite irrational in many aspects and do not follow rational prescriptions. In this paper, we build a Logic of Irrational and conflicting agents called I-Agent Logic of Uncertainty (IALU) as a vector logic of evaluations of sentences. This logic does not prescribe rules on how an agent should reason rationally, but describe rules on how agents reason irrationally. This is a descriptive not prescriptive theory in contrast with the classical logic and the probability theories. This provides a new possibility to better understand and model uncertainties associated with social conflict phenomena. We show that the fuzzy logic has a potential to become a scalar version of a descriptive logic of irrational agents because it satisfies several necessary conditions of IALU.*

1. INTRODUCTION

Real agents can be rational only to some extent and be quite *irrational* in many aspects. This leads to a fundamental difference in the way in which theories of uncertainties can justify their axioms. Some theories (logics) of uncertainties prescribe uncertainty values purely mathematically without experiments. We call such logics *prescriptive theories* of uncertainty. They tell how agent should assign uncertainty values to be rational.

Alternative agent logics of uncertainty depend on empirical, physical knowledge about real behavior of agents in specific contexts. These logics are *descriptive* -- they describe how agents actually produce and combine uncertainty values. Such logics cannot be built by pure mathematical axiomatic means, they need experiments or experience from the open real world.

This paper introduces a logic uncertainty theory in the context of conflicting evaluations of agent preferences. This includes logic rules and a mechanism for identifying types of logic relative to levels of conflict and self-conflicts. We show advantages of interpretation of fuzzy logic as an extension of the classical logic and probability theory for a mixture of conflicting and non-conflicting agents. This interpretation shows that fuzzy logic has potential to become a scalar version of a descriptive logic of irrational agents.

This work is a further development of our previous works [5-13] that contain extensive references to related work. The fundamental analysis of relevant issues can be found in [1-4]. In [2] agents are connected to logic and probability in the following way. In a given state s, the formula $P_j(\varphi)$ denotes the probability of the logic proposition φ according to the agent j's probability distribution in the state s. In this model, an individual agent gives the probability. All agents are autonomous, and no conflict among individual agents or self-conflict is modeled explicitly. This formalization does not provide tools to fuse the contradictory knowledge of individual agents to probabilities.

We show that the conflict among agents and self-conflict for each agent break the traditional invariance/symmetry of the contradiction and tautology for any transformation of the propositions known in the classical logic and the probability theory. Therefore, the contradiction and the tautology are not always invariant.

2. PRESCRIPTIVE THEORY VS. DESCRIPTIVE THEORY

A logic expression $F(p_1,p_2,....,p_n)$ is called *invariant* for the transformation h if

$$F(p_1,p_2,....,p_n) = F(h(p_1),h(p_2),....,h(p_n))$$

For example given the logic expression

$$F(p_1,p_2) = (p_1 \wedge \neg p_2) \vee (\neg p_1 \wedge p_2)$$

and the transformation $h(p_1) = p_2$, $h(p_2) = p_1$ we have

$$F(h(p_1),h(p_2)) = (p_2 \wedge \neg p_1) \vee (\neg p_2 \wedge p_1) = F(p_1,p_2)$$

Prescriptive uncertainty theories prescribe that

(P1) uncertainty P of any proposition p is evaluated by a single scalar $P(p)$ value,

(P2) the truth-value of any tautology $(p \vee \neg p)$ is true for every p, and

(P3) the truth-value of any contradiction $(p \wedge \neg p)$ is false for every p.

The *invariance (symmetry)* to the order of variables and their substitution is a fundamental property of tautology and contradictions in the classical logic. A relation $R\ (x,y)$ is *symmetrical* if and only if,

$$\forall\ x,y\ \ R(x,y)=R(y,x).$$

In the classical logic logical operations $\vee$ and $\wedge$ symmetrical to the *order* of variables

$$\forall\ x,\ y\quad (x \vee y = y \vee x)\quad \&\quad (x \wedge y = y \wedge x).$$

and are invariant (symmetrical) for *substitution* of p_i by any q_i, $h(p_1)=q_1$, $h(p_2) = q_2$

$$(p_1 \vee \neg p_1) = (q_1 \vee \neg q_1), \qquad (p_2 \wedge \neg p_2) = (q_2 \wedge \neg q_2).$$

There is also a known *anti-symmetry* in the classical logic -- any tautology is converted to a contradiction when we change the logic operator $\vee$ with $\wedge$ and vice verse (logic duality),

$$(p \vee \neg p) = \neg\ (p \wedge \neg p).$$

Irrational agents can break both classical symmetry and antisymmetry

$$(p_1 \vee \neg p_1) \neq (q_1 \vee \neg q_1),$$

that is for irrational agents $(p_1 \vee \neg p_1)$ can be tautology, but $(q_1 \vee \neg q_1)$ is not. Similarly, for irrational agents, $(p_1 \wedge \neg p_1)$ can be a contradiction but $(q_1 \vee \neg q_1)$ is not,

$$(p_1 \wedge \neg p_1) \neq (q_1 \wedge \neg q_1),$$

For instance symmetry $x \vee y = y \vee x$ may not be true for the irrational agent for the contradiction $(p_1 \wedge \neg p_1)$ with the following substitution $q_1 = \neg p_1$ and $\neg q_1 = p_1$

$$(p_1 \wedge \neg p_1) \neq (\neg p_1 \wedge p_1).$$

By breaking anti-symmetry

$$(p \vee \neg p) \neq \neg\ (p \wedge \neg p).$$

the irrational agent may state that $(p \vee \neg p)$ and $(p \wedge \neg p)$ both are true or both are false at the same time.

This irrationally leads to breaking prescription (P1) of using a *single number* for the evaluation, because p and $\neg$p are "independent" for an irrational agent. We may have situations where according to the irrational agent the tautology $(p \vee \neg p)$ has a vector value (F,T) not only True. Similarly, $(p \wedge \neg p)$ is not only False but has two values (T,F) denoted as T&F. The new mixed vector values F&T and T&F are possible for an irrational agent but impossible in the prescriptive logic of rational agents. These new logic vector values are in conflict with the classical logic.

The vector logic of irrational agents substitutes prescriptions such as (P1)-(P3) listed above by a description of agents evaluations in the vector form that include vector forms of classical T and F

$$True = \begin{bmatrix} True \\ True \end{bmatrix}, False = \begin{bmatrix} False \\ False \end{bmatrix}$$

and self-conflicting evaluations

$$T\ \&\ F = \begin{bmatrix} True \\ False \end{bmatrix}, T\ \&\ F = \begin{bmatrix} False \\ True \end{bmatrix},$$

Thus, the prescription rules such as classical tautology and contradiction become only descriptive rules. We may not know the value of the tautology and the contradiction for a specific irrational agent in advance, as we know for rational agents. We must only describe agent's vector truth values. for a particular situation.

3. LOGIC OF FIRST ORDER CONFLICTING AGENTS

Vector logic

To define a concept of first order conflicting agents we set up a framework using an example of agents who buy cars. The concept of first order conflicting agents is much wider than provided in the example with a binary preference relation. Using a preference relation we show that at the first order of conflicts AND and OR operations should not be traditional scalar classical logic operations, but **vector operations** to reflect a structure of individual agent evaluations.

Consider 100 agents g_i, two cars A and B and preference relation ">" between cars to be assigned by each of these agents (potential buyers). We define a Boolean variable X such that X=1 (True) if $A>B$ else X=0 (False). Each agent g_i answered a questionnaire with two options offered: (1) "$A>B$ is true" and (2) "$A>B$ is false". Say 70 agents marked "$A>B$ is true" giving a frequency value, $m(A>B)$= 70/100, that can be interpreted as a probability or fuzzy logic membership function value.

The situation for which a group of agents assumes that statement p="A>B" is true and a complementary group of agents assumes that the same statement A>B is false, is a situation of conflict/contradiction among agents that are wrestling for the logic value of A > B. We denote this situation as a **first order of conflict/contradiction**. Here each individual agent is completely *rational* and has *no self-conflict*. The conflict exists only between *different agents* g_i and g_j when they provide true/false evaluation v of the same statement p,

$$v_i(p) \neq v_j(p).$$

We specify a set of agents G, a subsets of agents $G(A>B)$ for which A > B and $G(A<B)$ for which A < B:

$$G(A>B) = \{g \in G \mid A >_g B \text{ is true}\},$$

$$G(A<B) = \{g \in G \mid A >_g B \text{ is false}\},$$

where $>_g$ is a preference relation by agent g. Sets $G(A>B)$ and $G(A<B)$ are complimentary, thus we will also use notation, $G^c(A>B)=G(B>A)$.

Example. Let G = {Agent$_1$, Agent$_2$, Agent$_3$, Agent$_4$} and $G(A>B)$= {Agent$_1$,Agent$_4$} and $G(A<B)$ = {Agent$_2$,Agent$_3$}. This is a conflicting set of agents. The evaluation for all four agents can be recorded in this way

$$value(A>B) = \begin{bmatrix} Agent_1 & Agent_2 & Agent_3 & Agent_4 \\ True & False & False & True \end{bmatrix}.$$

Similarly, for N agents we can record the evaluations as follows

$$V(A>B) = \begin{bmatrix} agent_1 & agent_2 & \ldots & agent_N \\ v_1(A>B) & v_2(A>B) & \ldots & v_N(A>B) \end{bmatrix}$$

where v_i (A > B) = T if agent g_i evaluates A > B as true and v_i (A > B) = F if agent g_i evaluates A > B as false. Now the previous evaluation for the N agents can be written as a vector

$$V(A>B)= (v(A>_1B), v(A>_2B),\ldots, v(A>_NB))=(v_1,\ldots,v_N),$$

where v_i is a short notation for $v(A>_iB)$. In general, vector

$$\boldsymbol{v} = (v_1,\ldots,v_N)$$

is a vector of T/F logic values given by N agents to any proposition p (not only p="A>B") with $v_i = v_i(p)$ as a truth value given to p by agent g_i.

Vector logic operations $\wedge, \vee, \neg$, **min** and **max** are defined as follows [14-17]:

$$\boldsymbol{v} \wedge \boldsymbol{u} = (v_1 \wedge u_1, \ldots, v_N \wedge u_N)$$

$$\boldsymbol{v} \vee \boldsymbol{u} = (v_1 \vee u_1, \ldots, v_N \vee u_N)$$

$$\neg \boldsymbol{v} = (\neg v_1, \ldots, \neg v_N)$$

$$\mathbf{min}\ (\boldsymbol{v}, \boldsymbol{u}) = (\min(v_1, u_1),\ldots,\min(v_N, u_N))$$

$$\mathbf{max}\ (\boldsymbol{v}, \boldsymbol{u}) = (\max(v_1, u_1),\ldots,\max(v_N, u_N)),$$

where the symbols $\wedge\ \vee\ \neg$ in the right side of equations are the classical scalar AND , OR and NOT operations.

Thus, in the first order of conflict the **vector logic operations** $\wedge, \vee, \neg$ are

$$V(p \wedge q) = \begin{bmatrix} agent_1 & \ldots & \ldots & agent_N \\ v_1(p) \wedge v_1(q) & \ldots & \ldots & v_N(p) \wedge v_N(q) \end{bmatrix} \quad (1)$$

$$V(p \vee q) = \begin{bmatrix} agent_1 & \ldots & \ldots & agent_N \\ v_1(p) \vee v_1(q) & \ldots & \ldots & v_N(p) \vee v_N(q) \end{bmatrix} \quad (2)$$

$$V(\neg p) = \begin{bmatrix} agent_1 & \ldots & \ldots & agent_N \\ \neg v_1(p) & \ldots & \ldots & \neg v_N(p) \end{bmatrix} \quad (3)$$

we remark that in this vector logic

$$\boldsymbol{v} \wedge \boldsymbol{u} = \mathbf{min}\ (\boldsymbol{v},\boldsymbol{u}), \quad \boldsymbol{v} \vee \boldsymbol{u} = \mathbf{max}\ (\boldsymbol{v},\boldsymbol{u})$$

that is

$$\begin{aligned} V(p \wedge q) &= \mathbf{min}(V(p), V(q)), \\ V(p \vee q) &= \mathbf{max}(V(p), V(q)) \end{aligned} \quad (4)$$

This vector logic is a descriptive theory for the first order of conflicting agents. Its advantage is that it describes the situation without loosing any information about evaluation (preferences) of individual agents and preserves the structure of these original evaluations. In contrast, commonly used scalar evaluations compress originals multidimensional information to a single number and looses a structure of evaluations and preferences.

Scalar logic

A traditional way to deal with contradictory evaluations of N agents provided in vector $\boldsymbol{v}$ is to compress (fuse) $\boldsymbol{v}$ to a scalar by computing a frequency of preferences of all agents as

$$\mu(A>B) = \frac{v(A>_1 B)+\ldots+v(A>_N B)}{N}$$

In general, for an arbitrary proposition p this means

$$\mu(p) = \frac{v_1(p)+\ldots+v_N(p)}{N}, \quad (5)$$

where $\mu(p)$ is the number of agents for which p is true.

Formula (5) describes uncertainty of evaluations of p in a generalized way, but it lost evaluations of individual agents. This leads to our inability to evaluate correctly μ $(p \wedge q)$ having only $\mu(p)$ and $\mu(q)$ using a fuzzy logic rule:

$$\mu(p \wedge q) = \min(\mu(p), \mu(q)) \quad (6)$$

Example. Let $\boldsymbol{v}$=(0,1,0,) given by three agents for p and $\boldsymbol{u}$=(1,0,1) given by the same agents for q. Thus

$$\boldsymbol{v} \wedge \boldsymbol{u} = (v_1 \wedge u_1, \ldots, v_N \wedge u_N) = (0,0,0)$$

For this vector (000) produced from $p \wedge q$, formula (5) provides $\mu(p \wedge q)$=0.

In contrast, formula (6) based m(p)=1/3 for $\boldsymbol{v}$=(0,1,0,) and μ(q)=1/3 for $\boldsymbol{u}$=(1,0,1) gives us

$$\mu(p \wedge q) = \min(\mu(p), \mu(q)) = 1/3.$$

This example shows how scalar uncertainty measures loose structural information of evaluations of individual agents and provide inaccurate measures of uncertainty of the situation. Thus, it is hard to justify such scalar measures for the descriptive theories of uncertainty because they do not describe correctly real preferences of agents and their evaluations. The problem of justification of such scalar evaluations for prescriptive theories is even harder. Why should it be prescribed? What is the norm that it enforces?

The vector operations (1)-(4) are context dependent in contrast with scalar operation (5). The advantage of these vector operations is that they preserve a structure of individual agent evaluations that is lost in the fuzzy logic (scalar logic).

4. SECOND ORDER OF CONFLICT

We denote a situation as a *higher order of conflict* if there are individual agents in a set of agents G that have self-conflict in addition to the possible first order conflict between different agents when they evaluate the same statement.

A *second order of conflict*/contradiction can take place if we have two complimentary evaluation criteria (e.g., for preferences objects). In general, we define an **n-th order of conflict** as a conflict that involves n complimentary evaluation criteria.

Mutual exclusion vs. ghost events. Implicitly, the classical logic and the probability theory assume the first order of conflict and rational agents. This means that every agent marks in the questionnaire event e_1 = "A > B is true" or an opposite event e_2 = "B > A is true", but not both of them. Both situations are possible but only one appears at the time. This is a fundamental assumption of the probability theory – elementary events are *disjoint, mutually exclusive (ME) and one of them needs to happen*. Agents must select only one of two events (e_1, e_2).

At the first order of conflict, we have a single logic value $v(p)$ that is True or False for each agent and statement p. For the second order of conflict assume that a set of agents G has two different binary *criteria*, or *channels* (C_1 and C_2) to evaluate the proposition p="A > B". Thus, each agent g has a set S of four possible evaluation states S = (T&T, T&F , F&T, F&F) obtained by the superposition of the two criteria, as it is shown below:

$$\begin{bmatrix} criterion(C_1) & T & T & F & F \\ criterion(C_2) & T & F & T & F \\ superposion & TT & T\&F & F\&T & FF \end{bmatrix}$$

We may have for agent g, the two mutually exclusive events/statements $e_i = p = (A >_g B)$ and $e_j = q = (B >_g A)$. If we introduce explicitly the two criteria C_1 and C_2 (instead of a single implicit C in the first order of conflict), we can define preference events in each criterion for agent g:

$$e_i = (A >_{C_1 g} B),\, e_j = (A >_{C_2 g} B\,),$$
$$e_i \,\&\, e_j = (A >_{C_1 g} B) \,\&\, (A >_{C_2 g} B\,)$$

We remark that the symbol "&" must not be confused with the logic symbol "$\wedge$". The symbol "&" means the superposition of True and False that can be also self conflicting as T&F , F&T. The symbol "$\wedge$" is the traditional conjunction operator where the conflict is absent.

For the irrational agent contradictory events e_i and e_j are not mutually exclusive. Thus, for the irrational agent event e_i&e_j exists. We call event e_i&e_j a *ghost event*. In terms of the T and F the notation e_i&e_j corresponds to a pair T&F, where criterion C_1 is True, but criterion C_2 is False for A>B. The fundamental advantage of the channel method is that we explicitly model evaluation criteria. However, this method has also an important disadvantage. We may not know explicit criteria and may not know that only two specific criteria C_1 and C_2 are involved in the evaluation.

Let agents g_1 and g_2 evaluated p using criteria C_1 and C_2 as follows,

$$V_1(p) = \begin{bmatrix} criterion\,(C_1\,) & T & T \\ criterion\,(C_2\,) & T & F \\ superposition & T & T\&F \end{bmatrix} \quad (7)$$

$$V_2(p) = \begin{bmatrix} criterion\,(C_1\,) & T & F \\ criterion\,(C_2\,) & T & T \\ superposition & T & F\&T \end{bmatrix}, \quad (8)$$

where the second column shows evaluations of p and the third column shows evaluations of $\neg$p. This gives us two different evaluations of the negation by agents g_1 and g_2

$$V_1(\neg p) = \begin{bmatrix} criterion\,(C_1\,) & F & F \\ criterion\,(C_2\,) & F & T \\ superposition & F & F\&T \end{bmatrix},$$
$$V_2(\neg p) = \begin{bmatrix} criterion\,(C_1\,) & F & T \\ criterion\,(C_2\,) & F & F \\ superposition & F & T\&F \end{bmatrix} \quad (9)$$

Agent g_1 has a self-conflict (T,T) in evaluating p and $\neg$p using criterion C_1 and has no self-conflict (T,F) in evaluating p and $\neg$p using C_2. In contrast, agent g_2 has no self-conflict (FT) in evaluating p and $\neg$p using criterion C_1 and has self-conflict (F,F) in evaluating p and $\neg$p using C_2.

To shorten notation we omit the first column and third row in $V(p)$ and use T for T&T and F for F&F,

$$V_1(p) = \begin{bmatrix} T & T \\ T & F \end{bmatrix}, \quad V_2(p) = \begin{bmatrix} T & F \\ T & T \end{bmatrix}, \qquad (10)$$

$$V_1(\neg p) = \begin{bmatrix} F & F \\ F & T \end{bmatrix}, V_2(\neg p) = \begin{bmatrix} F & T \\ F & F \end{bmatrix} \qquad (11)$$

Now we can explore (12), which is a contradiction in the classical logic because in classical logic if $V_1(p)$= T then $V_j(\neg \mathrm{p})$ = F for any agent g_j including g_2.

To transform (12) we use (10) and (11).

$$\begin{bmatrix} V_1(p) \wedge V_1(\neg p) & V_1(p) \wedge V_2(\neg p) \\ V_2(p) \wedge V_1(\neg p) & V_2(p) \wedge V_2(\neg p) \end{bmatrix} = \qquad (12)$$

$$\begin{bmatrix} \begin{bmatrix} T & T \\ T & F \end{bmatrix} \wedge \begin{bmatrix} F & F \\ F & T \end{bmatrix} & \begin{bmatrix} T & T \\ T & F \end{bmatrix} \wedge \begin{bmatrix} F & T \\ F & F \end{bmatrix} \\ \begin{bmatrix} T & F \\ T & T \end{bmatrix} \wedge \begin{bmatrix} F & F \\ F & T \end{bmatrix} & \begin{bmatrix} T & F \\ T & T \end{bmatrix} \wedge \begin{bmatrix} F & T \\ F & F \end{bmatrix} \end{bmatrix} =$$

$$\begin{bmatrix} \begin{bmatrix} F & F \\ F & F \end{bmatrix} & \begin{bmatrix} F & T \\ F & F \end{bmatrix} \\ \begin{bmatrix} F & F \\ F & T \end{bmatrix} & \begin{bmatrix} F & F \\ F & F \end{bmatrix} \end{bmatrix} = \begin{bmatrix} F & (F, T \,\&\, F) \\ (F, F \,\&\, T) & F \end{bmatrix} (13)$$

Thus, the self-conflicting vector evaluations $V_1(p)$ and $V_2(\neg p)$ of two agents g_1 and g_2 have produced a self-conflicting vector conjunction $V_1(p) \wedge V_2(\neg p)$.

Similarly, we have a self-conflicting disjunction

$$\begin{bmatrix} V_1(p) \vee V_1(\neg p) & V_1(p) \vee V_2(\neg p) \\ V_2(p) \vee V_1(\neg p) & V_2(p) \vee V_2(\neg p) \end{bmatrix} =$$

$$\begin{bmatrix} \begin{bmatrix} T & T \\ T & F \end{bmatrix} \vee \begin{bmatrix} F & F \\ F & T \end{bmatrix} & \begin{bmatrix} T & T \\ T & F \end{bmatrix} \vee \begin{bmatrix} F & T \\ F & F \end{bmatrix} \\ \begin{bmatrix} T & F \\ T & T \end{bmatrix} \vee \begin{bmatrix} F & F \\ F & T \end{bmatrix} & \begin{bmatrix} T & F \\ T & T \end{bmatrix} \vee \begin{bmatrix} F & T \\ F & F \end{bmatrix} \end{bmatrix} =$$

$$\begin{bmatrix} \begin{bmatrix} T & T \\ T & T \end{bmatrix} & \begin{bmatrix} T & T \\ T & F \end{bmatrix} \\ \begin{bmatrix} T & F \\ T & T \end{bmatrix} & \begin{bmatrix} T & T \\ T & T \end{bmatrix} \end{bmatrix} = \begin{bmatrix} T & (T, T \,\&\, F) \\ (T, F \,\&\, T) & T \end{bmatrix} \quad (14)$$

Thus, formulas (13) and (14) show that the two self–conflicting agents g_1 and g_2 that evaluated p irrationally as shown in (7) and (8) produce the self-conflicting conjunction and disjunction.

Example. Assume that a proposition p is true for three agents g_1,g_2 and g_3, false for two agents, g_4 and g_5, p=T&F for two agents g_6 and g_7 and p=F&T for agent g_8 out of the total 8 agents. For agents g_4 and g_5, p=T and p=F. If we interpret p=F is a classical logic way, we get $\neg p$=T and $p \wedge \neg p$ = T$\wedge$T=T by combining p=T and $\neg p$=T. Thus, the contradiction is true which is in complete discord with the classical logic. In a similar way we can get that the tautology $p \vee \neg p$ = F by considering agent g_8 with p=F and p = T. In the classical logic way we infer $\neg p$ = F from p = T and $p \vee \neg p$ = F $\wedge$ F=F . This is also in a complete discord with the classical logic.

Similarly fuzzy logic operations

$$\mu(p \wedge q) = \min[\mu(p), \mu(q)],$$
$$\mu(p \vee q) = \max[\mu(p), \mu(q)],$$
$$\mu(\neg p) = 1 - \mu(p)$$

produce

$$\mu(p \wedge \neg p) = \min[\mu(\mathrm{p}), 1 - \mu(\mathrm{p})],$$
$$\mu(p \vee \neg p) = \max[\mu(\mathrm{p}), 1 - \mu(\mathrm{p})],$$

which means that the value of contradiction, $\mu(p \wedge \neg p)$ can be greater than zero and the value of the tautology, $\mu(p \vee \neg p)$ can be less than one

$$\mu(p \wedge \neg p) > 0, \quad \mu(p \vee \neg p) < 1 \qquad (15).$$

Example: Consider six agents g_1,…g_6 that evaluated proposition p as follows:

$$\boldsymbol{v}(p) = (v_1(p), \ldots v_6(p)) = (\mathrm{T,T,T,F,F,T})$$

Similarly these agents evaluated $\boldsymbol{v}(\neg p)$,

$$\boldsymbol{v}(\neg p) = ((\mathrm{v}_1(\neg p), \ldots \mathrm{v}_6(\neg p)) = (\mathrm{T,T,F,F,T,T})$$

The probability theory approach will exclude all irrational agents keeping only two rational agents g_3 and g_5 with $P(p)$=2/2=1, $P(\neg p)$=0/2=0 for probabilities of p and $\neg$p, The use of formula (5) to compute frequencies without exclusion irrational agents produces:

$$\mu(p)=4/6, \; \mu(\neg p)=4/6=0, \; \mu(p \wedge \neg p)=3/6, \; \mu(p \vee \neg p)=5/6,$$

Thus $\mu(\mathrm{p} \wedge \neg \mathrm{p}) > 0$ and $\mu(\mathrm{p} \vee \neg \mathrm{p}) < 1$ which contradict to the classical logic and probability theory but are in accord with the fuzzy logic. In other words, in the fuzzy logic, the contradiction can have a non-zero degree of truth and the tautology can have a non-zero degree of falseness. This is completely in disagreement with the classical logic, where the tautology always is true and contradiction is always false. This property tells that fuzzy logic has a potential to become a *descriptive theory* for modeling self-conflicting irrational agents. At this moment, the major obstacle to reach this goal is that properties (15) are only necessary but are *not sufficient* conditions to be a descriptive theory of irrational agents.

We believe that the vector logic is adequate tool to model the second order of conflict (self-conflicting irrational agents). Any scalar measure μ can be appropriate only if it *contradicts insignificantly* the vector logic for a particular problem at hand. The elaboration of this criterion is a subject of the future research.

5. Conclusion

It is shown in this paper that the contradiction and the tautology are not always false for the presence of the vector logic values T&F and F&T. These logic values express conflicting and self-conflicting situations, which break the classical symmetry. Agents that use such vector logic values do not follow the classical logic for some propositions. This vector logic values introduces the possibility of constructing a logic of contradictory agents with contradictions that can be true and tautologies that can be false. This paper had shown that the space of vector evaluations for the first order of conflict (conflicts between agents) differs from this space for second order of conflict that includes self-conflict of individual agents.

We expect that, in the near future, the area of I-Agent Logic of Uncertainty (IALU) that studies how a mixture of rational, irrational, conflicting, or inconsistent agents reason, will be an active area for new fundamental research and discovery.

6. REFERENCES

[1] Carnap R., Jeffrey R, Studies in Inductive Logics and Probability, vol. 1, 35-165 Berkeley, CA, University of California Press, 1971.

[2] Fagin R., Halpern J. Reasoning about Knowledge and Probability, Journal of the ACM 41, 2 , 1994, 340 – 367.

[3] Halpern J. Reasoning about uncertainty, MIT Press, 2005.

[4] Hisdal E., Logical Structures for Representation of Knowledge and Uncertainty, Springer, 1998.

[5] Kovalerchuk B., Analysis of Gaines' logic of uncertainty, In: Proceeding of NAFIPS '90 vol.2 edited by I.B. Turksen, Toronto Canada pp.293-295, 1990.

[6] Kovalerchuk B., Context spaces as necessary frames for correct approximate reasoning. International Journal of General Systems, v.25, n 1, 1996, 61-80.

[7] Resconi G., J. Klir and U. St. Clair, Hierarchical uncertainty metatheory based upon modal logic, Int. J. of General Systems, vol. 21, pp. 23-50, 1992.

[8] Resconi G., Jain, L. Intelligent agents, Springer, 2004.

[9] Resconi G., Kovalerchuk, B., The Logic of Uncertainty with Irrational Agents In: Proc. of JCIS-2006 Advances in Intelligent Systems Research, Taiwan, Atlantis Press, 2006.

[10] Resconi G., I. B. Türkşen, Canonical forms of fuzzy truthoods by meta-theory based upon modal logic, Information Sciences, vol. 131, pp. 157– 194, 2001.

[11] Resconi G., B. Kovalerchuk , The Logic of Uncertainty with Irrational Agents , JCIS 2006

[12] Resconi, G., Agents and Lattice , ISKE 2006

[13] Resconi G., I.B.Turksen , Truthoods based on an Additive Semantic Measure with Break of Global Symmetry in modal logic, IJFS March 2006

[14] Mizraji, E., Vector logics: the matrix-vector representation of logical calculus. Fuzzy Sets and Systems, 50, 79–185, 1992.

[15] Mizraji, E. Modalities in vector logic. Notre Dame J. Formal Logic 35, no. 2 (1994), 272–283

[16] J. Westphal, J. Hardy, "Logic as a Vector System", Journal of Logic and Computation, vol. 15 no. 5, (2005) pp. 751-765.

[17] Radojević, D., Marić, Z. Vector Interpolative Logic, SISY 2005, www.bmf.hu/conferences/sisy2005/Radojevic.pdf