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Hierarchy of logics of irrational and conflicting agents

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Abstract.

This paper proposes a hierarchy of logics of agents relative to levels of their conflicts, self-conflicts and irrationality to provide a base for several studies on foundations of the theories of uncertainties. These studies include the foundation of known uncertainty theories (probability theory, fuzzy logic, and others) as well as new logics and types of uncertainties. Probability theory, fuzzy logic, and others theories of uncertainties provide a calculus for manipulating with probabilities, membership functions, and other types of uncertainty indicators. However, these theories lack a mechanism for getting initial (basic) uncertainties. The proposed hierarchy of conflicting and irrational agents creates a base for generating uncertainty values, logic operations with these values and for comparing different types of uncertainty for the preference relation. A core concept of this hierarchy is the concept of expansion by superposition that includes fusion and adjustment of contradictory events and statements.

Keywords: uncertainty logic, conflicting agent, irrational agent, probability theory, fuzzy logic, self-conflict, mutual exclusion.

1. Introduction

The concept of rational agents in [14] is motivated by fundamental goals in the *software engineering* area to provide a formal logicisation of software engineering [3]. In this area, the rationality of a software agent rests in the maximization of its chances of success based on software agent's knowledge of its environment. Any agent that maximizes chances of success is considered as a rational one. In fact, an agent's success criterion as well as the knowledge of the environment can be quite irrational. As a result, agent's behavior is not rational, thus, the concept of agent's rationality/irrationality should be developed deeper and more specifically in this area if software agent applications that model complex irrational human behavior are envisioned.

The research area that is deeply interested in partially rational agents called boundedly rational agents is motivated by fundamental goals in *psychology and economics* [5,10]. The works in this area explore the psychology of intuitive beliefs

and choices by examining their bounded rationality as Kahneman outlined in his Nobel Prize Lecture in 2002. In economics, bounded rationality means use of (1) multiple utility functions instead of one global scalar utility function, (2) limited types of utility functions due to cost of information collection, and (3) heuristics.

Our concept of irrational agent is motivated by fundamental goals in the *artificial intelligence* area of *logic and reasoning under uncertainty* [6,7] with agents [4]. We envision a formal logicisation of reasoning of irrational agents that do not follow rational reasoning in the frames of the classical logic and the probability theory and even fuzzy logic. This area is quite different from two previous areas as well as the areas that study expert reasoning simply because irrational agents hardly fit a definition of experts. We define a specific concept of an *ME-rational* (the agent that is rational relative to mutual exclusion) with a clear criterion how to distinguish such agents. Similarly, we define a concept of an *ME-irrational* agent as a *self-conflicting agent* as well as *contradictory agents* that contradict each other but without self-conflict in judgment. The concepts of conflict and self-conflict in judgments are critical for our study of irrational agents, because uncertainty often means that several contradictory statements are made. We build our approach on the results of direct measurements or answers of agents not on the aggregated utility functions to be able to model a structure of contradictions explicitly.

Conducting experiments with devices or asking agents are two common ways to assign initial (basic) uncertainties. We can ask agents to evaluate classical truth-value $v(p)$ of a statement p (true/false) and then compute frequencies of these answers provided by a set of agents G . Alternatively we can ask each agent to give us a truth-value $v(p)$ in the $[0,1]$ interval. Both frequency and the subjective evaluations then can be an empirical base for a probability of the statement p [1,6,7] or as a value of the fuzzy logic membership function (MF)[7]. An elaborated description of these processes treating MF in the probabilistic framework is given in [7]. Many other authors claim that MFs represent a type of uncertainty that differs from probabilistic one. However, an explicit model that supports this claim is absent. The goal of this paper is to build a conceptual base for answering this question in a rigorous way using the agent approach.

This paper introduces a hierarchy of logics of uncertainty in the context of conflicting evaluations of preferences by agents that includes a mechanism for identifying a type of logic and reasoning computations in the agent logic. It is a further development of our previous works [8-13] that contain extensive references to related work. The fundamental analysis and review of relevant issues can be found in [1-4].

The probability calculus does not incorporate explicitly the concepts of irrationality and conflicts of agents. It misses the structural information at the level of individual objects, but preserves the information on a global property of a set of objects. Given a dice the probability theory studies frequencies of the different faces $E=\{e\}$ as independent elements. This set of elementary events E has no structure. It is only required that elements of E are *mutually exclusive* and *complete* (no other alternative is possible). The order of its elements is irrelevant to probabilities of each number in E . No irrationality or conflict is allowed in this definition. The classical probability calculus does not provide a mechanism for modelling uncertainty when agents

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communicate (collaborates or conflict). Recent work by Halpern [6] is an important attempt to fill this gap.

Now we can provide a more formal definition. A reasoning agent g is called *totally consistent for statement* S if g always provides the same truth value for S , thus the change of context C and time t does not change S value for agent g . Note that total consistence does not imply correctness of the evaluation of S by g because a consistent truth-value can be incorrect. The classical logic models such agents and their reasoning. A reasoning agent g is called *partially consistent for statement* S if g *not* always provides the same truth-value for S . If sources of inconsistency are numerous, cannot be traced or explicitly presented then the probability theory is used commonly to model such agents and their reasoning with probability $P(S)=n/N$, where n is the number of cases when g answered that S is true and N is a total number of cases. Otherwise, the classical logic still can be used.

Similarly, a reasoning agent g is called *totally consistent for a set of statements* $\{S\}$ if g always provides the same truth value for each S from $\{S\}$. Again, the classical logic is applicable for such agent. A reasoning agent g is called *partially consistent for a set of statement* $\{S\}$ if g *not* always provides the same truth-value for some or all statements from $\{S\}$. The expansion from a single statement S to a set of statements $\{S\}$ can be done differently with *different relations among statements* in the $\{S\}$ for the agent g . In addition, set $\{S\}$ may contain statements about more than one object, event. Relations among statements and objects define applicability of the probability theory. Kolmogorov's axioms of the probability theory set up these relations.

One of the most important of these axioms is a mutual exclusion axiom for elementary events. Let E be a predicate which is true if e is an elementary event, $\{e\}$ be a set of all given elementary events, and $P(e)$ be a probability of event e , then $P(e_i \& e_j)=0$ and $P(e_i \vee e_j)=P(e_i)+P(e_j)$. Property $P(e_i \& e_j)=0$ means that e_i and e_j cannot happen simultaneously, Let $S(e) = \text{True}$ if elementary event e has happened. If g tells that e_i has happened, $S(e_i)=\text{True}$, then g should tell $S(e_j)=\text{False}$, i.e., $S(e_i) \& S(e_j)=\text{False}$ for this agent. We will call this *agent g rational on Mutual Exclusion (ME-rational agent)*. Accordingly we will call *agent g irrational on Mutual Exclusion (ME-irrational agent)* if g tells both $S(e_i)=\text{True}$ and $S(e_j)=\text{True}$ for mutually exclusive events e_i and e_j , $S(e_i) \& S(e_j)=\text{True}$. In other words, we can say that for this agent g contradiction $S(e_i) \& S(e_j)$ is True.

We distinguish between conflicting agents, self-conflicting agents, and irrational agents. Each individual agent can be very consistent, but agent g_1 can contradict agent g_2 creating a pair of **conflicting agents** (g_1, g_2) . A **self-conflicting agent** g can provide two contradictory preferences, $(p, \neg p)$ with $p=(A > B)$ and $\neg p=(B > A)$ for statement p by stating the p true and false at the same time. This can be an indication that g has two different competing criteria, C_1 and C_2 , such as less price and better quality. However, this does not resolve the ultimate preference contradiction it only explains it because the goal is to make a preference between A and B and the explanation does not solve this problem.

If criteria C_1 and C_2 are the same then we have an **irrational agent** g in a specific context. Thus, a set of irrational agents is a subset of the class of self-conflicting agents. The agent can be irrational because of inability to understand complex **context and evaluate criteria**. In the stock market, some traders quite often do not understand

a complex context and rational criteria of trading. These traders are irrational agents exhibiting chaotic, random, and impulsive behavior. At the same market situation they can sell and buy stocks, exhibiting irrational (p,¬p) behavior.

2. Framework of first order conflicting agents

To define a concept of first order conflicting agents we set up a framework using an example of agents who buy cars. The concept of first order conflicting agents is much wider than provided in the example with a binary preference relation. Using this preference relation we show that at the first order of conflicts AND and OR operations should not be traditional scalar classical logic operations, but **vector operations** to reflect a structure of individual agent evaluations.

Consider 100 agents g_i , two cars A and B and preference relation “ $>$ ” between cars to be assigned by each of these agents (potential buyers). We define a Boolean variable X such that $X=1$ (True) if $A>B$ else $X=0$ (False). Each agent g_i answered a questionnaire with two options offered: (1) “ $A>B$ is true” and (2) “ $A>B$ is false”. Say 70 agents marked “ $A>B$ is true” giving a frequency value, $m(A>B)=70/100$, that can be interpreted as a probability or fuzzy logic membership function value.

The situation for which a group of agents assumes that $A>B$ is true and a complementary group of agents assumes that the same $A>B$ is false, is a situation of conflict/contradiction among agents that are wrestling for the logic value of $A > B$. We denote this situation as a **first order of conflict/contradiction**. Here *each individual agent is completely rational and has no self-conflict. The conflict exists only between different agents when they evaluate the same statement.*

We specify a set of agents G , a subset of agents $G(A>B)$ for which $A > B$ and a set of agents $G_{A<B}$ for which $A < B$:

$$G(A>B) = \{g \in G \mid A >_g B \text{ is true}\}, \quad G(A<B) = \{g \in G \mid A >_g B \text{ is false}\},$$

where $>_g$ is a preference relation by agent g . Sets $G(A>B)$ and $G(A<B)$ are complimentary, thus we will also use notation, $G^c(A>B)=G(B>A)$.

Example. Let $G = \{\text{Agent}_1, \text{Agent}_2, \text{Agent}_3, \text{Agent}_4\}$ and $G(A>B) = \{\text{Agent}_1, \text{Agent}_4\}$ and $G(A<B) = \{\text{Agent}_2, \text{Agent}_3\}$. This is a conflicting set of agents. The evaluation for all four agents can be recorded in this way

$$\text{value}(A > B) = \begin{bmatrix} \text{Agent}_1 & \text{Agent}_2 & \text{Agent}_3 & \text{Agent}_4 \\ \text{True} & \text{False} & \text{False} & \text{True} \end{bmatrix}.$$

Similarly, for N agents we can record the evaluations as follows

$$V(A > B) = \begin{bmatrix} \text{agent}_1 & \text{agent}_2 & \dots & \text{agent}_N \\ v_1(A > B) & v_2(A > B) & \dots & v_N(A > B) \end{bmatrix}$$

where $v_i(A > B) = \text{true}$ if agent g_i evaluates $A > B$ as true and $v_i(A > B) = \text{false}$ if agent g_i evaluates $A > B$ as false. Now the previous evaluation for the four agents can be written as $V(A>B) = (v(A>_1B), v(A>_2B), v(A>_3B), v(A>_4B))$ or more generally for N agents $V(A>B) = (v(A>_1B), v(A>_2B), \dots, v(A>_NB))$.

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Below we show that at the first order of conflicts, AND and OR operations should not be the traditional scalar classical logic operations, but should be **vector operations** that can be aggregated to a scalar value in $[0,1]$ interval later if needed. In this way, operations reflect a structure of individual agent evaluations.

In fact, at the first order of agent conflict we must introduce a **fusion process** of the logic values given by the agents. A basic way to do this is to compute a frequency of preferences of all agents:

$$\mu(A > B) = \frac{v(A >_1 B) + \dots + v(A >_N B)}{N}.$$

An alternative way is to use affine weighted frequencies.

At first glance, $\mu(A > B)$ is the same as used in the probability and utility theories. However, classical axioms of the probability theory have no references to agents producing initial uncertainty values and do not violate the mutual exclusion. As a result, formulas for assigning initial uncertainty values such as $\mu(A > B)$ are not a part of the theory. Value $\mu(A > B)$ can differ significantly from the classical relative frequency for some irrational agents.

A probability value can be agent's judgment or a result of physical experiments. In [2] agents are connected to logic and probability in the following way. In a given state s , the formula $P_j(\varphi)$ denotes the probability of the logic proposition φ according to the agent j 's probability distribution in the state s . In this model, an individual agent gives the probability. All agents are autonomous and no conflict among agents and self-conflict is modeled explicitly. This formalization does not provide tools to fuse the contradictory knowledge of different agents.

In the first order of irrationality the **vector logic operations** are

$$\begin{aligned} V(p \wedge q) &= \begin{bmatrix} agent_1 & \dots & agent_N \\ v_1(p) \wedge v_1(q) & \dots & v_N(p) \wedge v_N(q) \end{bmatrix}, \\ V(p \vee q) &= \begin{bmatrix} agent_1 & \dots & agent_N \\ v_1(p) \vee v_1(q) & \dots & v_N(p) \vee v_N(q) \end{bmatrix}, \\ V(\neg p) &= \begin{bmatrix} agent_1 & \dots & agent_N \\ \neg v_1(p) & \dots & \neg v_N(p) \end{bmatrix}, \end{aligned}$$

where the symbols $\wedge \vee \neg$ in the right side of equations are the classical AND, OR and NOT operations. For a set of conflicting agents at the first order of conflicts, we have these important properties

$$\begin{aligned} G(p \wedge q) &= G(p) \cap G(q) = \min(G(p), G(q)) - G(\neg p \wedge q), \\ G(p \vee q) &= G(p) \cup G(q) = \max(G(p), G(q)) + G(\neg p \wedge q), \end{aligned}$$

where $G(p)$ is the set of agents for which statement p is true, $G(q)$ is the set of agents for which statement q is true. If $G(q) \subset G(p)$ then we have well known min, max operations. We remark also that $G(\neg p) = G^C(p)$, where $G^C(p)$ is the complementary set of $G(p)$ in G . Thus,

$$\begin{aligned} G(p \vee \neg p) &= G(p) \cup G(\neg p) = G(p) \cup G(\neg p) = G(p) \cup G(p)^C = G \\ G(p \wedge \neg p) &= G(p) \cap G(\neg p) = G(p) \cap G(p)^C = \emptyset \end{aligned}$$

In other words, $G(p \wedge \neg p) = \emptyset$ corresponds to the contradiction $p \wedge \neg p$ that is always false and $G(p \vee \neg p) = G$ corresponds to the tautology $p \vee \neg p$ that is always true in the first order of conflict.

In this section, we formalized the concept of first order conflicting agents with the conflict only between different agents while each agent has no self-conflict. It is shown that for these conflicts AND and OR operations should be defined as vector operations that preserve a structure of individual agent evaluations. Next, we had shown that a preference evaluation by multiple contradictory and self-contradictory agents (that contradict each other) can differ from a traditional frequency that does not preserve a structure of individual agent evaluations.

3. Second order of conflict, self-conflict, and irrationality

In this section, we use concepts of irrational agents to explain the deviation of the fuzzy logic from the classical logic and the probability theory where the contradiction is always false and the tautology is always true. We discovered that this deviation is a consequence of agent's self-conflict that is not allowed in the classical logic and the probability theory for mutual exclusion (ME). The existence of a self-conflicting ME-state for agents such as real people is the main motivation to introduce second order of conflict. We formalize this concept below.

To explain the deviation of fuzzy logic from the classical logic and the probability theory we introduce a new (for fuzzy logic and probability theory) type of evaluation (self-conflicting vector evaluation) as well as a superposition process to combine self-conflicting logic values. Below we define the concepts of a higher order of conflict including a concept of the a second order of conflict/contradiction. In general, we define an n -th order of conflict. The existence of self-conflicting states motivates changes in the definition of the negation operator and computations of contradiction and tautology that differ from the definitions in the classical logic.

Deviation from classical logic in fuzzy set theory. We denote a situation as a *higher order of conflict* if there are individual agents in a set of agents G that have self-conflict in addition to the possible first order conflict between different agents when they evaluate the same statement. a *second order of conflict/contradiction* can take place if we have two complimentary evaluation criteria (e.g., to set up preferences of a set of objects). In general, we define an **n -th order of conflict** as a conflict that involves n complimentary evaluation criteria.

Zadeh has defined fuzzy sets and logic operations by the formulas:

$$\begin{aligned}\mu(p \wedge q) &= \min[\mu(p), \mu(q)], \mu(p \vee q) = \max[\mu(p), \mu(q)], \\ \mu(\neg p) &= 1 - \mu(p).\end{aligned}$$

Thus, $\mu(p \wedge \neg p) = \min[\mu(p), 1 - \mu(p)]$, $\mu(p \vee \neg p) = \max[\mu(p), 1 - \mu(p)]$. Therefore, here the value for contradiction, $\mu(p \wedge \neg p)$ can be greater than zero and for the tautology, $\mu(p \vee \neg p)$ can be less than one. In other words, in the fuzzy logic, the contradiction can have a non-zero degree of truth and the tautology can have a non-zero degree of falseness. This is completely in disagreement with the classical logic, where the tautology always is true and contradiction is always false. Below we

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present a possible explanation of these fuzzy logic properties by using the concept of self-conflicting agents at the second order. However this explanation does not justify specific min, max operations and negation, $\mu(\neg p) = 1 - \mu(p)$ used in fuzzy logic.

Mutual exclusion vs. ghost events. Implicitly, the classical logic and the probability theory assume that agents are *rational*, that is every agent marks in the questionnaire even $e_1 = "A > B \text{ is true}"$ or an opposite event $e_2 = "B > A \text{ is true}"$, but not both of them. Both situations are possible but only one appear at the time. This is a fundamental assumption of the probability theory – elementary events are *disjoint, mutually exclusive (ME) and one of them needs to happen*. Agents must select only one of two **optional???** options (e_1, e_2).

In general, if an agent marks both options (case (c) below), both answers will be discarded, as non-valid produced by an ME-irrational agent and only ME-rational agents with a single answer will be counted in relative frequencies.

☒ A>B ☐ B>A	☐ A>B ☒ B>A	☒ A>B ☒ B>A
(a) Rational agent selected event $e_1 = "A > B"$	(b) Rational agent selected event $e_2 = "B > A"$	(c) Irrational agent selected $e_1 \& e_2$.

Events e_1 and e_2 can differ from preferences between A and B. They can be any mutually exclusive events as shown below. We will call event $e_1 \& e_2$ a **ghost event**, it does not exist in a set of mutually exclusive alternatives $\{e_1, e_2\}$, but an ME-irrational agent who marks events randomly or deliberately irrationally creates this ghost event.

☒ e_1 ☐ $e_2 = \neg e_1$	☐ e_1 ☒ $e_2 = \neg e_1$	☒ e_1 ☒ $e_2 = \neg e_1$
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How to deal with ME-irrational agents? There are three approaches: (1) ignoring ME-irrational agents by excluding their answers if we have only a few ME-irrational agents, (2) redesigning the questionnaire (change events) to satisfy mutual exclusion to be able to use the probability theory, and (3) developing a new logic to deal with a mixture of ME-irrational agents and rational agents for situations when redesign is not feasible. Redesigning a set of events often is not feasible because for any given set of events we do not know in advance how many agents are ME-irrational for these events. Thus, potentially a redesign process can continue indefinitely. The same question: "How many agents are ME-irrational for these events?" can be raised after every consequent redesign. Therefore, it is desirable to develop a *theory to deal with a mixture* of rational agents.

Expansion by superposition. Now we will explore the issues that determine the possibility of such a theory. The first idea is that we discussed above was to add a new alternative $e_i \& e_j$ to two mutually exclusive events e_i and e_j , that is recognizing that e_i and e_j can be more than just mutually exclusive in some situations.

We call this method an **expansion by superposition** of a set of events where two mutually exclusive events e_i and e_j are mixed or superposed with possible adjustment of meaning of e_i and e_j . In general, this method does not explain why seemingly mutually exclusive alternatives can coexist. It can range from an irrational belief of the agent in such ghost events, or be similar to the superposing situation in quantum

physics. In addition, there are cases where a simple explanation is possible. For instance, if e_i ="young" and e_j ="old" then a new event $e_i \blacksquare e_j$ can be called "middle-age" with adjustment of e_i being new e_i "young" that does not include "middle-age". Similarly, adjusted "old" does not include "middle-age." For a relevant discussion, see [11,12]. The **superposition operation** $\blacksquare$ is a new operation that produces a new event/statement $e_i \blacksquare e_j$, which fuses events e_i and e_j .

Channel method. Assume that a set of agents G has two different binary criteria, or channels (C_1 and C_2) to evaluate the sentence " $A > B$ ". Thus, each agent has a set S of four possible evaluation states $S = (TT, T\&F, F\&T, FF)$ obtained by the superposition of the two criteria, as it is shown below:

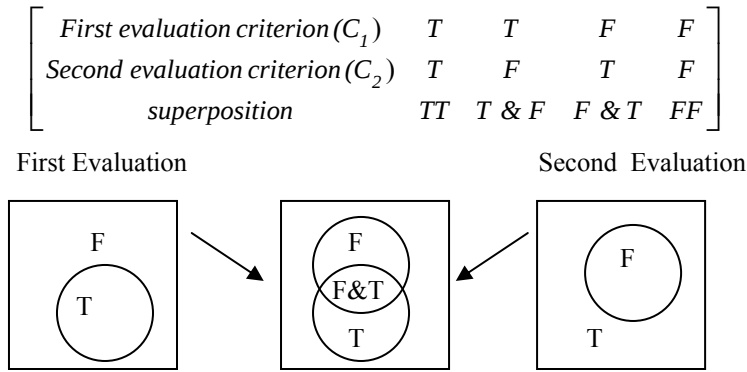


Fig. 1. Second order of conflict as a superposition of two first order of conflict types of evaluations for two internal criteria of the agent in the same time and space.

In terms of events e_i and e_j with a single criteria C (that is not explicitly given or not given at all) we may have for agent g two mutually exclusive events/statements $e_i = p = (A >_g B)$ and $e_j = q = (B >_g A)$. If we introduce explicitly two criteria C_1 and C_2 (instead of implicit C) we can define preference events in each criterion for agent g : $e_i = (A >_{C_1g} B)$, $e_j = (B >_{C_2g} A)$, $e_i \wedge e_j = (A >_{C_1g} B) \wedge (B >_{C_2g} A)$

Now e_i and e_j are not mutually exclusive but still contradictory to be able to prefer A or B . In terms of T and F notation $e_i \& e_j$ corresponds to a pair TF , where criterion C_1 is True, but criterion C_2 is False for $A > B$. The fundamental advantage of the channel method is that we explicitly model evaluation criteria. However, this method has also an important disadvantage. We may not know explicit criteria and may not know that only two specific criteria C_1 and C_2 are involved in evaluation.

Negation operation. The negation operation at the second order of conflict requires special consideration starting from the definition of the truth of the statement. There are two truth values for each statement p , $V(p, C_1)$ and $V(p, C_2)$ for criteria C_1 and C_2 $V(p, C_1)$, respectively:

$$T = \begin{bmatrix} T \\ T \end{bmatrix}, F = \begin{bmatrix} F \\ F \end{bmatrix}, T \& F = \begin{bmatrix} T \\ F \end{bmatrix}, F \& T = \begin{bmatrix} F \\ T \end{bmatrix}$$

We can define p is true if $V(p, C_1) = V(p, C_2) = \text{True}$. The negation of p is defined similarly as $V(p, C_1) = V(p, C_2) = \text{False}$. Thus, we define a set of agent $G(p)$, such that p is true for both criteria C_1 and C_2 ,

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$$G(p) \equiv \{g \mid V(p, C_1) = V(p, C_2) = T\}$$

Similarly we define a set of agents $G(\text{NOT } p)$ for **negation** of p :

$$G(\neg p) \equiv \{g \mid V(p, C_1) = V(p, C_2) = F\}$$

These sets are equivalent to sets that we defined above, $G(p) = G_{TT}$ and $G(\text{NOT } p) = G_{FF}$.

An **alternative negation** is to use a complement set to the set

$G(p)$:

$$G(\text{NOT } p) \equiv G^c(p) = G_{TF} \cup G_{FT} \cup G_{FF}$$

This NOT operator differs from “ $\neg$ ” operator for the second order of contradiction (see also Figure 2), but these negations are equal in the first order of contradiction.

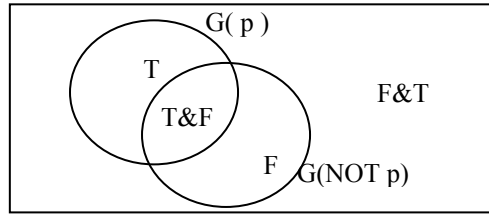


Fig. 2 Set of Agents for which NOT p is True.

Now we can clarify the concept of **irrational agents**. A **necessary condition** for agent g to be an irrational agent is $g \in G_{TF} \cup G_{FT}$.

A **sufficient condition** of agent g to be irrational is that criteria C_1 and C_2 are the same. Thus, cases FT and TF are equal. Agent g is called **seemingly irrational** if criteria C_1 and C_2 are not identified but two contradictory evaluations are provided by the agent g for a statement p . A set of agents that are irrational for p will be denoted as $G_I(p)$. Now with the second negation NOT we have these properties

$$G(p \wedge \text{NOT } p) = G(p) \cap G(\text{NOT } p) = \min(G(p), G(\text{NOT } p)) - G(\neg p \wedge \text{NOT } p)$$

$$G(p \vee \text{NOT } p) = G(p) \cup G(\text{NOT } p) = \max(G(p), G(\text{NOT } p)) + G(\neg p \wedge \text{NOT } p),$$

where $G(\text{NOT } p) \subset G(p)$.

Thus, for the second negation the set of agents in a contradiction state or in the state T & F (TF for short) is not an empty set. Given a set of agents $G(p)$, we can compute membership value μ by using the averaging operation used for computing frequencies, but these frequencies will be different from computed in the probability calculus, because counting evaluation of irrational agents. This approach can be generalized to define the third (and higher) order of conflicting agents with three or more criteria (channels) available to an agent to evaluate the same sentence.

4. Conclusion

The hierarchy of sets of agents to levels of conflicts, self-conflicts, and irrationality proposed in this paper is intended to provide a base for several areas of further studies. These studies include the foundation of probability theory, fuzzy logic, and other types of uncertainties, as well as real-world interpretation of these theories.

The major weakness in fuzzy logic is its ad hoc nature with operations that are not justified in advance (as it is done in the probability theory), but adjusted in many different ways (by using many different t-norms and t-conorms). The novelty of this paper is not in introduction of new formulas for initial membership function values and operations with them, but in creating a base for their *interpretation* by means of conflicting and irrational agents as an internal part of the theory. Physics provides plenty of positive examples of such interpretations that can be inspiration examples for building comprehensive logics of uncertainty for agents. We expect that in near future this will be an active area of new fundamental research and discoveries.

A society of agents always has a degree of conceptual conflicts and any agent can be self-conflicting. This affects agents' abilities to use resources for identifying goals and acting. In this paper, we established the fundamental logic structure that can be used for making logic decisions in a conflicting multi-agent environment.

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