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# Fusion in Agent –Based Uncertainty Theory and Neural Image of Uncertainty

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Abstract— **In neural network modeling, the goal often is to get a most specific crisp output (e.g., binary classification of objects) from neuron inputs that have multiple possible values. In this paper, we change the viewpoint and assume that the neuron is an operator that transforms binary classical logic input to the many valued logic output, e.g., changes crisp sets into fuzzy sets. In this interpretation, the neural network is composed of agents or neurons, which work to implement uncertainty calculus and many valued logics from crisp perceptual input. This idea is closely related to the Dynamic Logic approach and recent cognitive science experimental discoveries. According to this model having crisp perceptual input, brain (1) produces a less certain representation, (2) processes input at this uncertainty level of representation, (3) converts results to the next more certain level of information representation, (4) processes this information and (5) repeats these steps several times until the acceptable level of certainty is reached. To build such model we rely not on the binary logic but on the logic of the uncertainty to obtain the high flexibility and logic adaptation of the described process. This paper presents a concept of the Agent–based Uncertainty Theory (AUT) based on complex fusion of crisp conflicting judgments of agents Communication among agents is modeled by the fusion process in the neural elaboration.**

## I. Introduction

This paper presents a concept of the Agent–based Uncertainty Theory (AUT) based on complex fusion of crisp conflicting judgments of agents. Communication among agents is modeled by the fusion process in the neural elaboration. This theory is needed to elaborate the dynamic neural network model (DNNM) that having crisp perceptual input, (1) produces a less certain representation, (2) processes input at this uncertainty level of representation, (3) converts results to the next more certain level of information representation, (4) processes this information, and (5) repeats this steps several times until the acceptable level of certainty of the result is reached.

This idea is closely related to the Dynamic Logic approach [12-14], Adaptive Resonance Theory [15], and recent cognitive science experimental discoveries [12,17-19]. The neurological and fMRI neuroimaging had shown that conscious perceptions are preceded by activation of cortex areas, where top-down signals originate from memories-models. Also, initial top-down signals (models) are driven by "low-spatial frequency," that is *vague* contents of bottom-up signals [12].

The increase of uncertainty of knowledge representation is associated with increased amount of inconsistent, conflicting, and self-conflicting statements that are communicated in the network and used in the reasoning. The logic of uncertainty must to deal with such information. It should contain: (1) a classification and a hierarchy of the levels of uncertainty and contradiction of statements and (2) the logic to reasoning with them to be able to iterate DNNL process.

An agent that maximizes chances of success without any logical conflict is considered as a *rational agent*. In fact, an agent's success criterion as well as the knowledge of the environment can be quite conflicting from the logic viewpoint. The nature of the conflict must be explained given a context with objects and associated attributes.

Traditionally the context is a single world and no other world is possible. Now assume that any agent has its world and assigns a value to the attributes from its point of view or world. When the values are internal to the agent and other agents cannot know the value, we are in the same position as in the modal logic with a set of worlds (one for each agent without possibility to fuse the worlds).

However, when all agents know the values of others agents because we have only one world, this generates confusion and logic inconsistence. How is it possible that the same object in the same context can have an attribute (property) that is true and is false at the same time? The fusion of the evaluations of different agent worlds in a single world or context world is the source of semantic uncertainty. In this situation, the set of agents can be in a logical state that we can define as *irrational*.

The irrationality can be internal for any agent when different criteria or worlds are used by the same agent to give the logic value. When the agent fuses all of these criteria to a single criterion, the agent can enter to an irrational situation where confusion and semantic uncertainty are possible. Accordingly, agent's behavior can be irrational. Thus, the concept of agents with logic conflict and irrationality should be developed deeper and more specifically.

The research area that is deeply interested in partially rational agents called *boundedly rational agents* is motivated by fundamental goals in psychology and economics [3,4]. The works in this area explore the psychology of intuitive beliefs and choices by examining their bounded rationality

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as Kahneman outlined in his Nobel Prize Lecture in 2002. In economics, bounded rationality means use of (1) multiple utility functions instead of one global scalar utility function, (2) limited types of utility functions due to cost of information collection, and (3) heuristics.

Our concept of agent in a conflicting (irrational) logical state is motivated by fundamental goals in area of *reasoning under uncertainty* [5],[6] with agents [7]. The processing data at a higher level of uncertainty and with less specified objects has a natural link with the *biological principle of effort minimization* (EM) [16].

We envision a formal logic of reasoning of irrational agents that do not follow rational reasoning in the frames of the classical logic and the probability theory and even fuzzy logic. This area is quite different from two previous areas as well as the areas that study expert reasoning simply because irrational agents hardly fit a definition of experts.

We define a specific concept of an *ME-rational* (the agent that is rational relative to mutual exclusion) with a clear criterion how to distinguish such agents.

Similarly, we define a concept of an *ME-irrational* agent as a *self-conflicting agent* as well as *contradictory agents* that contradict each other but without self-conflict in judgment.

The concepts of conflict and self-conflict in judgments are critical for our study of irrational agents, because uncertainty often means that several contradictory statements are made. The concept of self-conflict itself is studied in paraconsistent logics [8] that is concern about the reasoning with contradictory statements.

We build our approach on the results of direct measurements or answers of agents not on the aggregated utility functions to be able to model a structure of contradictions explicitly.

This approach considers a hierarchy of logics of uncertainty in the context of conflicting evaluations of preferences by agents that includes a mechanism for identifying a type of logic and reasoning computations in the agent logic. We call this hierarchy an **Agent-based Uncertainty Theory (AUT)**. It is a further development of our previous works [9], [10] that contain extensive references to related work.

The probability calculus does not incorporate explicitly the concepts of irrationality or agent state of logic conflict. It misses the structural information at the level of individual objects, but preserves the information on a global property of a set of objects. Given a dice the probability theory studies frequencies of the different faces E={e} as independent elements. This set of elementary events E has no structure. It is only required that elements of E are *mutually exclusive* and *complete*, that is no other alternative is possible. The order of its elements is irrelevant to probabilities of each number in E. No irrationality or conflict is allowed in this definition.

The classical probability calculus does not provide a mechanism for modeling uncertainty when individual agents communicate and collaborates and fuse different individual logic values for attributes or properties of the same object in only one world. Recent work by Halpern [6] is an important attempt to fill this gap.

Now we can provide a more formal definition. A reasoning agent g is called *totally consistent for statement* S if g always provides the same truth value for S, thus the change of context C and time t does not change S value for agent g. Note that total consistence does not imply correctness of the evaluation of S by g because a consistent truth-value can be incorrect. Such agents and their reasoning are modeled in the classical logic.

A reasoning agent g is called *partially consistent for statement* S if g *not* always provides the same truth-value for S. If sources of inconsistency are numerous, cannot be traced or explicitly presented then the probability theory is used commonly to model such agents and their reasoning with probability P(S)=n/N, where n is the number of cases when g answered that S is true and N is a total number of cases. Otherwise, the classical logic still can be used.

Similarly, a reasoning agent g is called *totally consistent for a set of statements* {S} if g always provides the same truth value for each S from {S}. Again, the classical logic is applicable for such agent.

A reasoning agent g is called *partially consistent for a set of statement* {S} if g *not* always provides the same truth-value for some or all statements from {S}. The expansion from a single statement S to a set of statements {S} can be done differently with *different relations among statements* in the {S} for the agent g. In addition, set {S} may contain statements about more than one object, event. Relations among statements and objects define applicability of the probability theory. Kolmogorov's axioms of the probability theory set up these relations.

Each individual agent can be very consistent, but agent $g_1$ can contradict agent $g_2$ creating a pair of fused conflicting agents $(g_1,g_2)$. We distinguish between a state of logic conflict *among agents*, a state of logic conflict *inside the agent*. A self-conflicting agent g is in a *synchronic state* of logic conflict where logic values true and false interfere and are fused one with the other in a complex way in the same time. This can be an indication that g has two or more different competing criteria, $C_1, C_2,\ldots C_N$ , e.g., $C_1$ minimizes price and $C_2$ maximizes quality. Thus, a set of agents in a state of logic conflict can be a subset of the class of self-conflicting agents.

The agent can be in a logic conflict state because of inability to understand complex context and evaluate criteria. In the stock market, some traders quite often do not understand a complex context and rational criteria of trading and fuse conflicting situation in one. A logical structure of self-conflicting states and agents is much more complex than without self-conflict. In the case of N binary criteria $C_1$ , $C_2,\ldots C_N$ that evaluate the logic value for the same attribute, there are $2^N$ possible states and $2^N$ - 1 of them can exhibit conflict between criteria. There is only one vector of values of the criteria that has no contradiction for sure, (T,T,..,T).

## II. First order of conflict of logic states

To define a *first order conflicting agents* we set up a framework and use an example of agents who buy cars. Note that the concept of first order conflicting agents is much wider than provided in the example that deals with a binary preference relation. This example shows that at the first order of conflicts AND and OR operations should not be traditional classical logic operations, but operations in a vector space of the agents on where at any point we have a binary string of the evaluations for the same object and the same attributes.

We remark that in any string, each individual bit appears as a separate one, but in fact, such bits can represent a single fused evaluation. Thus, we may not able to separate the parts inside of the string because the *structure* of the bits carries the evaluation. For example, while a protein consists of different atoms, but we cannot separate them because the protein' property is produced by all atoms in a particular structure.

Similarly, aggregation of agents can mean much more than a collection of independent evaluations. It may reflect a conflicting structure of evaluations by self-coherent population of agents.

The situation for which a group of agents assumes that A>B is true and a complementary group of agents assumes that the same A>B is false, is a situation of conflict/contradiction among agents that are wrestling for the logic value of A > B.

We denote this situation as a **first order of conflict of logic state**. Here each individual agent is completely rational and has *no self-conflict*. The conflict exists only between *different agents* when they evaluate the same statement with the criteria C.

Example. Let $G = \{Agent_1, Agent_2, Agent_3, Agent_4\}$ and

$$G(A>B) = \{Agent_1, Agent_4\}$$

and

$$G(A<B) = \{Agent_2, Agent_3\}$$

The set G is in a conflict logic state. The evaluation for all four agents and the criteria C = " > " can be recorded in this vector inside the space of the agents

$$value(A > B) = \begin{bmatrix} Agent_1 & Agent_2 & Agent_3 & Agent_4 \\ True & False & False & True \end{bmatrix}.$$

Similarly, for N agents we can record the evaluations as follows

$$V(A > B) = \begin{bmatrix} agent_1 & agent_2 & \ldots & agent_N \\ v_1(A > B) & v_2(A > B) & \ldots & v_N(A > B) \end{bmatrix}$$

In a graphic way, this can be represented in the agent space as shown in Figure 1.

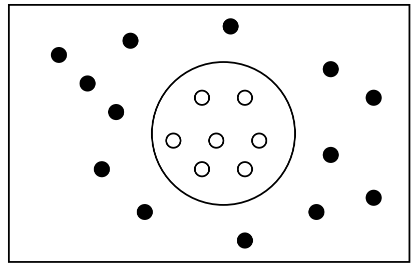

Figure 1. 20 agents in the logic conflicting state, where 7 white agents are in the state "true" and 13 black agents are in the state "false" for the same preference criteria C = ">".

At the first order of agent conflict, we must introduce a **fusion process** of the logic values given by the agents. A fused evaluation is a vague evaluation that reflects contradiction in evaluations.

A basic way to do this is to compute the frequency (or the weighted frequency) of preferences of all agents for the sentence

$$p = \text{"}A > B\text{"}$$

is

$$\mu(p) = w_1 v_1(p) + \ldots + w_N v_N(p) = \begin{bmatrix} v_1(p) \\ v_2(p) \\ \ldots \\ v_N(p) \end{bmatrix}^T \begin{bmatrix} w_1 \\ w_2 \\ \ldots \\ w_N \end{bmatrix} \qquad (1)$$

where $\begin{bmatrix} v_1(p) \\ v_2(p) \\ \ldots \\ v_N(p) \end{bmatrix}$ and $\begin{bmatrix} w_1 \\ w_2 \\ \ldots \\ w_N \end{bmatrix}$ are two vectors in the space of the agents' evaluations. The first vector contains all logic states (true/false) for all the agents, the second vector had a property

$$\sum_{k=1}^{N} w_k = 1$$

and contains a neural image of the logic fusion in AUT (see Figure 2 as an illustration).

Figure 2 can also be interpreted for a single self-conflicting agent, when $i = j = k$, but $v_i$, $v_j$ and $v_k$ are produced not by a single criterion C, but by different criteria $C_i$, $C_j$, and $C_k$ explicitly (consciously) or implicitly (unconsciously).

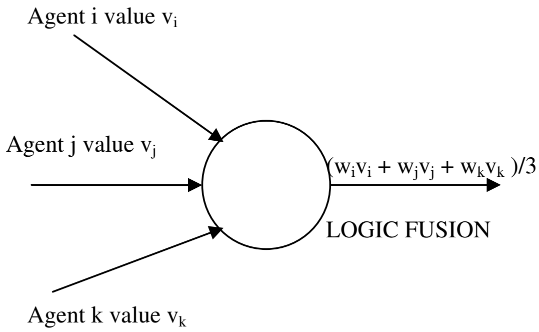


Figure 2. Neural image of the logic fusion in AUT system

In the first order of conflict, logic states of the agent space with **vector logic operations** are defined as

$$V(p \wedge q) = \begin{bmatrix} agent_1 & \dots & \dots & agent_N \\ v_1(p) \wedge v_1(q) & \dots & \dots & v_N(p) \wedge v_N(q) \end{bmatrix},$$

$$V(p \vee q) = \begin{bmatrix} agent_1 & \dots & \dots & agent_N \\ v_1(p) \vee v_1(q) & \dots & \dots & v_N(p) \vee v_N(q) \end{bmatrix},$$

$$V(\neg p) = \begin{bmatrix} agent_1 & \dots & \dots & agent_N \\ \neg v_1(p) & \dots & \dots & \neg v_N(p) \end{bmatrix},$$

where the symbols $\wedge$, $\vee$, $\neg$ in the right side of the equations are the classical AND , OR, and NOT operations.

Let $|G(p \wedge q)|$ and $|G(p \vee q)|$ be the numbers of agents for which $(p \wedge q)$ and $(p \vee q)$ are true respectively, then for a set of conflicting agents at the first order of conflicts, we following important properties take place

$$|G(p \wedge q)| = \min(|G(p)|, |G(q)|) - |G(\neg p \wedge q)|,$$

$$|G(p \vee q)| = \max(|G(p)|, |G(q)|) + |G(\neg p \wedge q)|,$$

where $|G(p)|$ and $|G(q)|$ are the numbers of respective agents for which statement p and q are true, respectively. If $G(q) \subset G(p)$ then we have well known min, max operations. Note that $G(\neg p) = G^C(p)$, where $G^C(p)$ is the complementary set of G(p) in G. Thus,

$$G(p \vee \neg p) = G(p) \cup G(\neg p) = G(p) \cup G(p)^C = G$$

$$G(p \wedge \neg p) = G(p) \cap G(\neg p) = G(p) \cap G(p)^C = \varnothing$$

In other words, $G(p \wedge \neg p) = \varnothing$ corresponds to the contradiction $p \wedge \neg p$ that is always false and $G(p \vee \neg p) = G$ corresponds to the tautology $p \vee \neg p$ that is always true in the first order conflict. If one of the sets G(p) and G(q) includes each other then

$$|G(p \wedge q)| = |G(p) \cap G(q)| = \min(|G(p)|, |G(q)|)$$

$$|G(p \vee q)| = |G(p) \cup G(q)| = \max(|G(p)|, |G(q)|)$$

In this case nothing is added or subtracted in

$$|G(p \wedge q)| = \min(|G(p)|, |G(q)|) - |G(\neg p \wedge q)|,$$

$$|G(p \vee q)| = \max(|G(p)|, |G(q)|) + |G(\neg p \wedge q)|.$$

## III. Example of Assigning Weights to Agents with and without Self-Conflict

Consider an example for assigning weights, where five agents provide their preference relation between objects A and C, say cars as shown in Table 1. The first four agents prefer A and the fifth agent prefer C with FR=5/4=0.8. Thus, we have an overwhelming preference of A.

In addition, we know truth-values of agents' preference relations A>B and B>C also shown in Table 1. Can this information help us in fusing preferences A>B? Will it change the fused preference or its confidence? Will it help to assign different weights to different agents and use a weighted frequency?

To answer these questions we added a line in Table 1 that indicates presence of self-conflict in agent's evaluations. We have three agents ($g_1$, $g_2$ and $g_5$) that have no self-conflict and agents $g_3$, $g_4$ have self-conflict.

We derived this conclusion from analysis of the transitivity of preferences by each agent. For instance, for $g_1$ we have A>B & B>C $\Rightarrow$ A>C, (i.e., A > B > C) and for $g_5$ the opposite is true: A < B , B < C $\Rightarrow$ A < C (i.e., C> B > A). Both cases are consistent with the transitivity of the preference relation.

Agent $g_2$ having B > A and B > C concludes that A > C that is B > A > C. This means that $g_2$ has no self-conflict. Agent $g_3$ having A > B and C > A concludes that A > C. This contradicts transitivity C > A > B). This means that G3 has self-conflict. Agent $g_4$ having A < B and B < C concludes A>C which violates transitivity and exhibits self-conflict.

If we rely only on most consistent agents ($g_1$, $g_2$ and $g_5$) we will have two opposite preferences A > C and C > A with frequency of A > C equal to 2/3=0.66 which is less then original 0.8.

To avoid removing self-conflicting agents we can assign them weights equal to 0.5 and have $EW=(1\cdot1+1\cdot1+0.5\cdot1+0.5\cdot1+1\cdot0)/5=3/5=0.6$ for A > C. If most of the agents are self-conflicting, then removing them can corrupt the fusion result and it will not represent agents' preferences.

This example shows that fusion of evaluations of different agents should not ignore information about agent's self-conflict status. Assigning weights is a simple way to do this. A more sophisticated way can be developed by a deeper analysis of types of self-conflicts and inter-agent conflicts.

TABLE 1.
PREFERENCE RELATIONS

| | Agent $g_1$ | Agent $g_2$ | Agent $g_3$ | Agent $g_4$ | Agent $g_5$ | FR | EW |
|---|---|---|---|---|---|---|---|
| A>C | 1 | 1 | 1 | 1 | 0 | 0.8 | 0.6 |
| A>B | 1 | 0 | 1 | 0 | 0 | 0.4 | |
| B>C | 1 | 1 | 0 | 0 | 0 | 0.4 | |
| Self-conflict | no | no | yes | yes | no | | |

## IV. Comparison with Paraconsistent Logics

First we summarize paraconsistent logics described in [8] in more detail. The main idea of da Costa's two-valued paraconsistent logic system $C_i$ [11] is that ~*A* is independent of *A* (both can be true at the same time). In addition, one of statements A or ~*A* must be true (the law of excluded middle), and if ~~*A* is true then *A* is true too. Further postulates constrain differently truth of ~(*A* & ~*A*) in different versions of $C_i$ system.

A three-valued logic is paraconsistent if it allows both a formula and its negation to be designated, Here *true* (only) and both *truth and false* are designated values out of three truth values: *true* (only) and *false* (only), and *both truth and false*.

A four-valued logic is defined similarly with the fourth value, *neither true nor false,* which is known as Dunn' s semantics for First Degree Entailment in relevant logics. A logic with infinite number of values in [0,1] with a suitable set of designated values is paraconsistent fuzzy logic.

All these logics have semantics in terms of possible worlds; say *A* is true in world *w*. However, they do not go further to identify how the truth-values are produced (measured and computed) in the world.

The proposed agent-based approach fills this important gap with the agent-based vector logic approach presented above. In particular, it shows that fusion does not mean and require the conversion to the scalar evaluations and operations. Moreover, for some types of conflicts fusion to scalars have no meaning.

## V. Example of a Neural Network with a Mixture of Irrational and Rational Agents

Figure 3 shows the neural network of five agents, where agents $g_1$-$g_4$ are rational agents and agent $g_5$ is an irrational agent. Each agent is modeled by its neural network (NN) with a crisp vector of inputs (**A**,**B**,**C**), where

**A** = $(a_1,a_2,\ldots a_n)$, **B** = $(b_1,b_2,\ldots b_n)$, **C** = $(c_1,c_2,\ldots c_n)$.

The NN can implement a logical judgment of rational agent $g_i$ by a classical modeling in NN a multidimensional Boolean function of (**A**,**B**,**C**). This Each agent $g_i$ from $g_1$-$g_4$ has two input vectors **A** and **B,** and scalar criterion $C_i$ that is implemented in the neural network, which computes the output C(**A**,**B**). The rationality of the agents $g_1$-$g_4$ follows from the fact that there is no any contradiction of its output with input data.

In contrast, an irrational agent $g_5$ has such conflict. It assigns negative and zero weights to correct inputs and finally outputs the judgment A<B, which has no rational confirmation. This agent ignores some evidence ($w_2=w_3=0$) and does not trust other evidence ($w_1=-1$). As a result, the decision is based only on the judgment by agent $g_4$.

Other sets of weights can model other types of irrationality and different fusions of rational judgments.

Agents g1-g4 can also behave irrationally. Let us consider an example of selection a car to buy using the following criteria that are not fully certain:

C1 Cost of the car (It could be the sell price, the price with maintenance or lease cost)

C2 Miles per gallon (It could be on highways, in town or in rural area)

C4 Comfort (It could be about low noise, adjustable chairs, heated chairs, CD or DVD.)

C4 Control quality (This could be about the panel design, lights, the navigator, or others.)

Assume that (1) agent g tells that car A is better than car B, (A>B) making no reference to the any specific criterion ($C_1$,$C_2$,$C_3$, $C_4$), and (2) the agent made this judgment by randomly picking up say criterion $C_1$ (cost of car) from $C_1$-$C_4$, that is A> B. For cars B and C the agent picks up $C_1$ again and says that B>C, but for cars A and C the agent uses criterion $C_2$ and says that C>A. This violates transitivity because the agent never mentioned (or even new consciously) any selection criterion or their mixture. The agent used irrational fusion as an economy principle [4].

Such irrational fusions can be explained by the principles of the *resource economy* [4] that is based on the biological principle of *effort minimization* [15] because it allows to simplify decision strategy and to avoid complex comparing contradictory alternatives. On the other hand, this fusion allows *modeling the principles* of resource economy/effort minimization. It also can be used to explore the effect of the evolution of perception of the irrational decisions by the irrational agents to be able to survive in the in the hostile environment

## VI. Conclusion

The major weakness in several uncertainty theories such as fuzzy logic is its ad hoc nature with operations that are not justified in advance (as it is done in the probability theory), but adjusted in many different ways (by using many different t-norms and t-conorms).

This paper presented a concept of the Agent–based Uncertainty Theory based on complex fusion of crisp conflicting judgments of agents. The AUT provides a uniform representation and an operational empirical interpretation for several uncertainty theories such as rough set theory, fuzzy sets theory, evidence theory, and probability theory. To build such uniformity the AUT exploits the fact that agents as independent entities can give conflicting evaluations of the same attribute. Conflicting evaluation are only active or generate uncertainty when the evaluations of the agents are fused in the same context of object evaluations. When evaluations are separated as in the modal logic (a single evaluation for a world) without fusion of the worlds there is no logical uncertainty.

The novelty of this paper is not in introduction of new formulas for initial membership function values and operations with them, but in creating a base for their interpretation by means of fusion of evaluations of conflicting and irrational agents as an internal part of the theory. A similar advantage the proposed approach has relative to paraconsistent logics.

In AUT, a world may have different evaluations for the same attribute given by different agents that are fused together and produce a fused evaluation that can be vague.

This is very far from the classic logic approach. We argue that such logic fusion for different types of conflicts is the model for many-value logics, fuzzy logic, neural fusion at the synapse and others.

In the proposed dynamic neural network model that uses AUT, neurons are many valued logic systems that can provide a new class of models for modeling the brain processes. The new neurons can perform many valued logic operations as inferential many valued operation as well as fuzzy logic and general uncertainty inferential operations by applying these operations. Ordinary neurons perform useful functions, the AUT neuron expand these functions to be a many value logic simulator by fusion.

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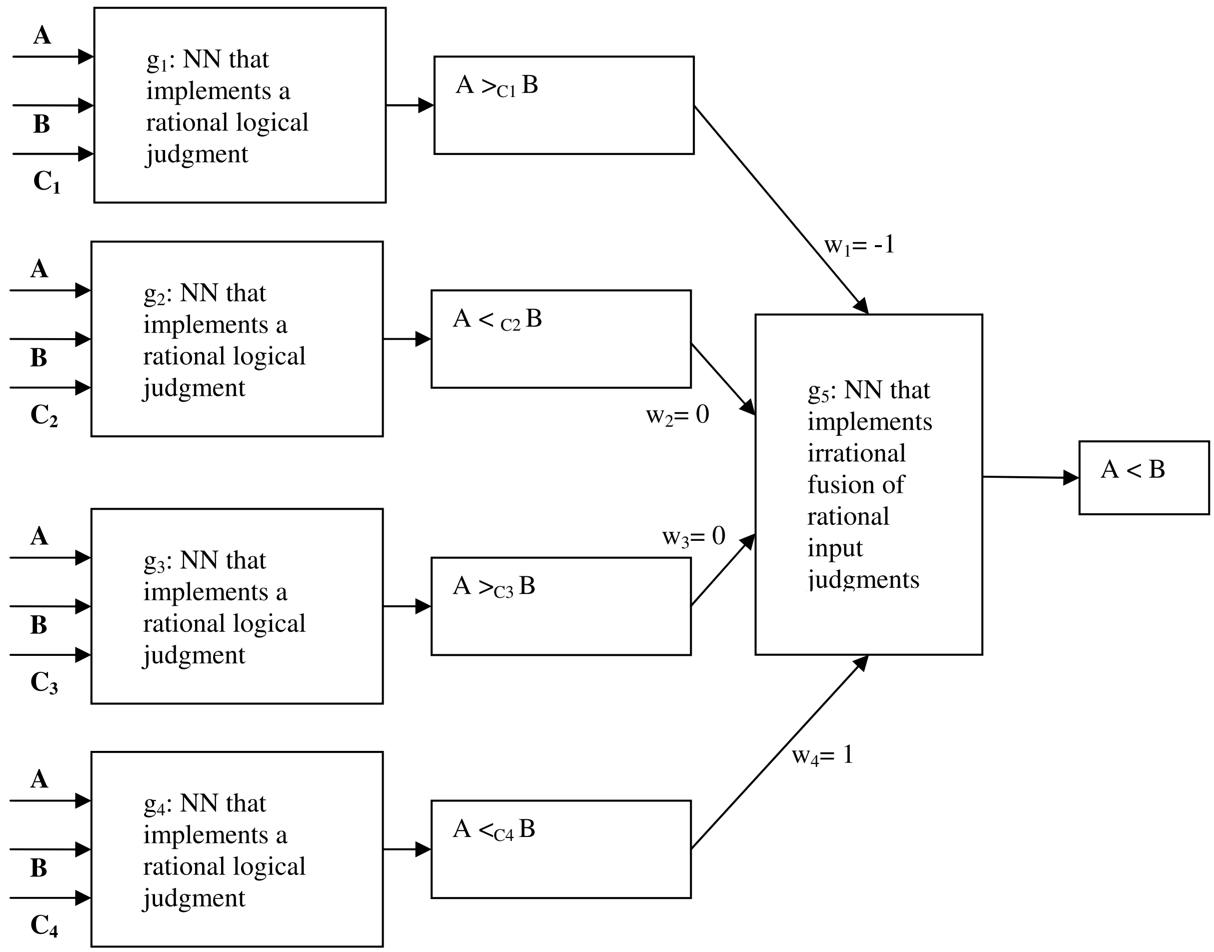


Figure 3. A neural network with rational agents $g_1$-$g_4$ and irrational agent $g_5$.