

Agents in Neural Uncertainty

Germano Resconi, Boris Kovalerchuk

Abstract—This paper models neural uncertainty using a concept of the Agent-based Uncertainty Theory (AUT). The AUT is based on complex fusion of crisp (non-fuzzy) conflicting judgments of agents. It provides a uniform representation and an operational empirical interpretation for several uncertainty theories such as rough set theory, fuzzy sets theory, evidence theory, and probability theory. The AUT models conflicting evaluations that are fused in the same evaluation context. This paper shows that the neural fusion at the synapse can be modeled by the AUT. The neuron is modeled as an operator that transforms classical logic expressions into many-valued logic expressions. The new neural network has neurons at two layers. The first-layer agents implement the classical logic operations, but at the second level, neurons or nagents (neuron agents) compute the same logic expression with different results for different agent inputs. The motivation for such neural network is to provide high flexibility and logic adaptation of the neural model.

I. INTRODUCTION

NEURAL networks integrate information from multiple neurons. Each neuron passes integrated information obtained from its inputs. The source of the input information and many important details of the input data typically are lost in such integration. The agent approach presented in this paper allows us to preserve this information and use it at the later steps of NN processing.

Uncertainty modeling is the area of extensive research with many advances and open problems [1],[2], [5]-[7], [11]-[19], [21]-[25], [33]-[34]. The agent-based modeling is another area of intensive current research [3]-[6], [20], [28]-[32]. In this paper, we model neural uncertainty using the Agent-based Uncertainty Theory (AUT) [35]. AUT exploits the fact that agents as independent entities can give conflicting evaluations of the same attribute. It models conflicting evaluations that are fused in the same evaluation context. If only one evaluation is allowed for each statement in each context (world) as in the modal logic [13] then there is no logical uncertainty. The situation is that the AUT models are inconsistent (fuzzy) and it is very far from the situation that is modeled by the traditional logic that assumes consistency. By incorporating such inconsistent statements, AUT is able to model different types of conflicts and their fusion known in many-valued logics, fuzzy logic, probability theory and other theories.

This paper shows that the neural fusion at the synapse can be modeled by the AUT. Agents in the neural network are represented by logic input values in the neuron itself. In the ordinary neural networks, any neuron is a processor that models a Boolean function. We change the point of view and consider a neuron as an operator that transforms classical logic expressions into many-valued logic expressions or in other words, changes crisp sets into fuzzy sets. This neural network consists of neurons at two layers. At the first one, neurons or agents implement the classical logic operations. At the second layer neurons or nagents (neuron agents) compute the same logic expression with different results. These are many-valued neurons that fuse results provided by different agents at the first layer. They fuse conflicting or inconsistent situations. The network is based on use of the logic of the uncertainty instead of the classical logic. The motivation for such neural network is to provide high flexibility and logic adaptation of the brain model. In this brain model, communication among agents is specified by the fusion process in the neural elaboration.

The probability calculus does not incorporate explicitly the concepts of irrationality or logic conflict of agent's state. It misses structural information at the level of individual objects, but preserves global information at the level of a set of objects. Given a dice the probability theory studies frequencies of the different faces $E=\{e\}$ as independent (elementary) events. This set of elementary events E has *no structure*. It is only required that elements of E are *mutually exclusive* and *complete*, that is no other alternative is possible. The order of its elements is irrelevant to probabilities of each element of E . No irrationality or conflict is allowed in this definition relative to mutual exclusion. The classical probability calculus does not provide a mechanism for modeling uncertainty when agents communicate (collaborates or conflict). Recent work by Halpern [6] is an important attempt to fill this gap.

This paper is organized as follows: Sections 2 and 3 provide a summary of the AUT starting from concepts and definitions. Section 4 presents a fusion process in the AUT. Section 5 discusses the neural images of the AUT. Section 6 provides an example of applications and section 7 concludes this paper.

II. CONCEPTS AND DEFINITIONS

Consider a set of agents $G=\{g_1, g_2, \dots, g_n\}$. Each agent g_k assigns binary true/false value $v \in \{\text{True}, \text{false}\}$ to proposition p . To show that v was assigned by the agent g_k we use notation $g_k(p) = v_k$.

Manuscript received January 5, 2009.

Germano Resconi is with the Department of Mathematics and Physics, Catholic University Brescia, I-25121, Italy, (e-mail: resconi@numerica.it).

Boris Kovalerchuk is with the Department of Computer Science, Central Washington University, Ellensburg, WA 98926-7520, USA (phone: 509-963-1438; fax: 509-963-1449; e-mail: borisk@cwu.edu).

Definition. An agent g is called a *reasoning agent* if g assigns a truth-value $v(p)$ to any proposition p from a set of propositions S and any logical formula based on S .

Definition. A set of reasoning agents G is called *totally consistent* for proposition p if any agent g from $\{g\}$ always provides the same truth value for p .

Thus, changing the agent does not change logic value $v(p)$ for a totally consistent set of agents. Here $v(p)$ is *global* for G and has *no local variability* that is independent on the individual agent's evaluation. The classical logic is applicable for such set of consistent (*rational*) agents.

Definition. A set of reasoning agents G is called *inconsistent* for proposition p if there are two subset of agents G_1, G_2 such that agents from them provides different truth values for p .

Definition. A set of reasoning agents G is called *S-only-consistent* if agents $\{g\}$ are consistent only for propositions in $S=\{p_1, p_2, \dots, p_n\}$ and are inconsistent in the *complimentary set* $\neg S = \{\neg p_1, \neg p_2, \dots, \neg p_n\}$.

The evaluations of p is a vector-function $v(p)=(v_1(p), v_2(p), \dots, v_n(p))$ for a set of agents G that we record as:

$$v(p) = \begin{pmatrix} g_1 & g_2 & \dots & g_{n-1} & g_n \\ v_1 & v_2 & \dots & v_{n-1} & v_n \end{pmatrix} \quad (1)$$

An example of the conflicting logic evaluations by five agent is shown below for $p = "A > B"$

$$f(p) = \begin{pmatrix} g_1 & g_2 & g_3 & g_4 & g_5 \\ true & false & true & false & true \end{pmatrix}$$

Kolmogorov's axioms of the probability theory are based on a totally consistent set of agents (for a set of statements) on mutual exclusion of elementary events. It follows from the definitions below.

Definition. Set $E=\{e_1, e_2, \dots, e_n\}$ is called a set of *elementary events* (or mutually exclusive events) for predicate E if

$$\forall e_i, e_j \in A \quad E(e_i) \vee E(e_j) = \text{True} \quad \text{and} \quad E(e_i) \wedge E(e_j) = \text{False}.$$

In other words, event e_i is an *elementary event* if for any $j, j \neq i$ events e_i and e_j cannot happen simultaneously, that is probability $P(e_i \wedge e_j) = 0$ and $P(e_i \vee e_j) = P(e_i) + P(e_j)$. Property $P(e_i \wedge e_j) = 0$ is the *mutual exclusion axiom* (ME- axiom).

Let $S=\{p_1, p_2, \dots, p_n\}$ be a set of statements, where $p_i=p(e_i)=\text{True}$ if and only if event e_i is an elementary event. In probability theory, $p(e_i)$ is not associated with **any** specific agent. It is assumed a global property (applicable to all agents). In other words, statements $p(e_i)$ are totally consistent for all agents. We associate p with each agent g and write $p_g(e_i)$, which indicates that it is assigned by agent g .

Definition. Let E_g be set of elementary events for agent g , then agent g is called a *ME-rational* agent if

$$\forall e_i, e_j \in A, p_g(e_i) \vee p_g(e_j) = \text{True}, p_g(e_i) \wedge p_g(e_j) = \text{False}.$$

In other words, these agents are *totally consistent* or *rational on Mutual Exclusion*. Previously we assumed that each agent assigns value $v(p)$ and we did not model this process explicitly. Now we introduce a set of criteria $C=\{C_1, C_2, \dots, C_m\}$ by which an agent can decide if a proposition p is true or false, i.e., now $v(p) = v(p, C_i)$, which means that p is evaluated by using the criterion C_i .

Definition. Given a set of criteria C , agent g is in a *self-conflicting state* if

$$\exists C_i, C_j (C_i, C_j \in C) \quad \& \quad v(p, C_i) \neq v(p, C_j)$$

In other words, an agent is in a self-conflicting state if two criteria exist such that p is true for one of them and false for another one. With the explicit set of criteria, the logic evaluation function $v(p)$ is not vector-function any more, but it is expanded to be a matrix function as shown below:

$$v(p) = \begin{bmatrix} & g_1 & g_2 & \dots & g_n \\ C_1 & v_{1,1} & v_{1,2} & \dots & v_{1,n} \\ C_2 & v_{2,1} & v_{2,2} & \dots & v_{2,n} \\ \dots & \dots & \dots & \dots & \dots \\ C_m & v_{m,1} & v_{m,2} & \dots & v_{m,n} \end{bmatrix} \quad (2)$$

Note that introduction of a set of criteria C explains self-conflict, but does not remove it.

If agent g can modify criteria in C making them consistent for p then g can resolve self-conflict. The agent can be in a logic conflict state because of inability to understand the complex *context* and to evaluate *criteria*.

A logical structure of self-conflicting states and agents is much more complex than it is without self-conflict. For m binary criteria $C_1, C_2, \dots, C_m$ that evaluate the logic value for the same attribute, there are 2^m possible states and only two of them (all true or all false values) do not exhibit conflict between criteria.

III. FIRST ORDER OF CONFLICT LOGIC STATE

Definition. A set of agents G is in a first order of conflicting logic state (*first order conflict*, for short) if

$$\exists g_i, g_j (g_i, g_j \in G) \quad \& \quad v(p, g_i) \neq v(p, g_j).$$

Definition. A set of agents G is in a **First Order Conflict (FOC)** for proposition p if

$$G(p) \cap G(\neg p) = \emptyset, \quad G(p) \neq \emptyset, \quad G(p) \cup G(\neg p) = G,$$

where $G(p)$ and $G(\neg p)$ are subsets of agents from G for which, respectively, $p, \neg p$ is true.

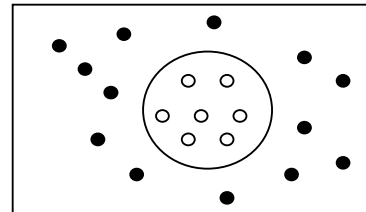


Fig. 1. A set of 20 agents in the first order of logic conflict.

Fig. 1 shows a set of 20 agents in the logic conflicting state, where 7 white agents are in the state True, and 13 black agents are in the state False for the same proposition $p = "A > B"$.

Below we show that at the first order of conflicts, AND and OR operations should differ from the classical logic operations and should be **vector operations** in the space of the agents' evaluations (*agents space*). The vector operations reflect a structure of logic conflict among coherent individual agent evaluations.

IV. FUSION PROCESS

If a single decision must be made at the first order of conflict, then we must introduce a **fusion process** of the logic values of proposition p given by all agents. A basic way to do this is to compute the weighted frequency of logic value given by all agents:

$$\mu(p) = w_1 v_1(p) + \dots + w_n v_n(p) = \begin{bmatrix} v_1(p) \\ v_2(p) \\ \dots \\ v_n(p) \end{bmatrix}^T \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{bmatrix} \quad (3)$$

where $\begin{bmatrix} v_1(p) \\ v_2(p) \\ \dots \\ v_n(p) \end{bmatrix}$ and $\begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{bmatrix}$ are two vectors in the space

of the agents' evaluations. The first vector contains all logic states (True/False) for all agents, the second vector

(with property $\sum_{k=1}^n w_k = 1$) contains non-negative weights

(utilities) that are given to each agent in the fusion process. In a simple frequency case, each weight is $1/n$. At first glance, $\mu(p)$ is the same as used in the probability and utility theories. However, classical axioms of the probability theory have no references to agents producing initial uncertainty values and do not violate the mutual exclusion. Next, we define **vector logic operations** for the first order of conflict logic states.

Definition

$$\begin{aligned} v(p \wedge q) &= v_1(p) \wedge v_1(q), \dots, v_n(p) \wedge v_n(q), \\ v(p \vee q) &= v_1(p) \vee v_1(q), \dots, v_n(p) \vee v_n(q) \\ v(\neg p) &= \neg v_1(p), \dots, \neg v_n(p), \end{aligned}$$

where symbols $\wedge$, $\vee$, $\neg$ in the right side of the equations are the classical AND, OR, and NOT operations.

These operations can be written with the explicit indication of agents. The first row indicates agents.

$$\begin{aligned} v(p \wedge q) &= \begin{pmatrix} g_1 & \dots & g_n \\ v_1(p) \wedge v_1(q) & \dots & v_n(p) \wedge v_n(q) \end{pmatrix} \\ v(p \vee q) &= \begin{pmatrix} g_1 & \dots & g_n \\ v_1(p) \vee v_1(q) & \dots & v_n(p) \vee v_n(q) \end{pmatrix} \\ v(\neg p) &= \begin{pmatrix} g_1 & \dots & g_n \\ v_1(\neg p) & \dots & v_n(\neg p) \end{pmatrix} \end{aligned}$$

Sets of conflicting agents at the first order of conflicts have important properties presented below. Let $|G(x)|$ be the numbers of agents for which proposition x is true.

Statement 1. If G is a set of agents at the first order of conflicts and $|G(q)| \leq |G(p)|$ then

$$|G(p \wedge q)| = \min(|G(p)|, |G(q)|) - |G(\neg p \wedge q)|, \quad (4)$$

$$|G(p \vee q)| = \max(|G(p)|, |G(q)|) + |G(\neg p \wedge q)|. \quad (5)$$

If G is a set of agents at the first order of conflicts and $|G(p)| \leq |G(q)|$ then

$$\begin{aligned} |G(p \wedge q)| &= \min(|G(p)|, |G(q)|) - |G(\neg q \wedge p)|, \\ |G(p \vee q)| &= \max(|G(p)|, |G(q)|) + |G(\neg q \wedge p)|. \end{aligned}$$

and also

$$|G(p \vee q)| = |G(p) \cup G(q)|, \quad |G(p \wedge q)| = |G(p) \cap G(q)|$$

Corollary 1. If G is a set of agents at the first order of conflicts such that $G(q) \subset G(p)$ or $G(p) \subset G(q)$ then

$$G(\neg p \wedge q) = \emptyset \text{ or } G(\neg q \wedge p) = \emptyset$$

$$|G(p \wedge q)| = \min(|G(p)|, |G(q)|)$$

$$|G(p \vee q)| = \max(|G(p)|, |G(q)|)$$

This follows from the statement 1. The corollary presents a well-known condition when the use of min, max operations has the clear justification. Let $G^c(p)$ is a *complement* of $G(p)$ in G :

$$G^c(p) = G \setminus G(p), \quad G = G(p) \cup G^c(p).$$

Statement 2. $G = G(p) \cup G^c(p) = G(p) \cup G(\neg p)$.

Corollary 2. $G(\neg p) = G^c(p)$

It follows directly from Statement 2.

Statement 3. If G is a set of agents at the first order of conflicts then

$$\begin{aligned} G(p \vee \neg p) &= G(p) \cup G(\neg p) = G(p) \cup G^c(p) = G \\ G(p \wedge \neg p) &= G(p) \cap G(\neg p) = G(p) \cap G^c(p) = \emptyset \end{aligned}$$

It follows from the definition of the first order of conflict and statement 2. In other words, $G(p \wedge \neg p) = \emptyset$ corresponds to the contradiction $p \wedge \neg p$, that is always false and $G(p \vee \neg p) = G$ corresponds to the tautology $p \vee \neg p$, that is always true in the first order conflict.

Let $G_1 \oplus G_2$ be a *symmetric difference* of sets of agents G_1 and G_2 ,

$$G_1 \oplus G_2 = (G_1 \cap G_2^c) \cup (G_1^c \cap G_2)$$

and let $p \oplus q$ be the *exclusive or* of propositions p and q ,

$$p \oplus q = (p \wedge \neg q) \vee (\neg p \wedge q).$$

Consider, a set of agents $G(p \oplus q)$. It consists of agents for which values of p and q differ from each other, that is

$$G(p \oplus q) = G((p \wedge \neg q) \vee (\neg p \wedge q)).$$

Below we use the number of agents in set $G(p \oplus q)$ to define a measure of difference between statements p and q and a measure of difference between sets of agents $G(p)$ and $G(q)$.

Definition. A measure of difference $D(p, q)$ between statements p and q and a measure of difference

$D(G(p), G(q))$ between sets of agents $G(p)$ and $G(q)$ are defined as follows:

$$D(p, q) = D(G(p), G(q)) = |G(p) \oplus G(q)|$$

Statement 4. $D(p, q) = D(G(p), G(q))$ is a distance, i.e., it satisfies distance axioms

$$\begin{aligned} D(p, q) &\geq 0 \\ D(p, q) &= D(q, p) \\ D(p, q) + D(q, h) &\geq D(p, h). \end{aligned}$$

This follows from the properties of the symmetric difference $\oplus$ [23].

Fig. 2 illustrates a set of agents $G(p)$ for which p is true and a set of agents $G(q)$ for which q is true. In Fig. 2(a) the number of agents for which truth values of p and q are different, $(\neg p \wedge q) \vee (p \wedge \neg q)$, is equal to 2. These agents are represented by white squares. Therefore, the distance between $G(p)$ and $G(q)$ is 2. Fig. 2(b) shows other $G(p)$ and $G(q)$ sets with the number of the agents for which $\neg p \wedge q \vee (p \wedge \neg q)$ is true equal to 6 (agents shown as white squares and squares with the grid). Thus, the distance between the two sets is 6.

In Fig. 2(a), set 2 consists of 2 agents $|G((p \wedge \neg q))| = 2$ and set 4 is empty, $|G((\neg p \wedge \neg q))| = 0$, thus $D(\text{Set2}, \text{Set4}) = 2$. This emptiness means that a set of agents with true p includes a set of agents with true q . In Fig. 2(b), set 2 consists of 4 agents $|G((p \wedge \neg q))| = 4$ and set 4 consists of 2 agents, $|G((\neg p \wedge \neg q))| = 2$, thus $D(\text{Set2}, \text{Set4}) = 6$.

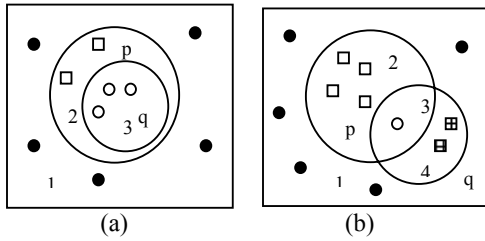


Fig. 2. A set of total 10 agents with two different splits to $G(p)$ and $G(q)$ subsets (a) and (b). These splits produce different distances between $G(p)$ and $G(q)$. The distance in the case (a) is equal to 2, the distance in case (b) is equal to 6. Set 1 (black circles) consists of agents for which both p and q are false, Set 2 (white squares) consists of agents for which p is true but q is false. Set 3 (white circles) consists of agents for which p and q are true, and Set 4 (squares with grids) consists of agents for which p is false and q is true.

For complex computations of logic values provided by the agents, we can use a *graph of the distances* among the sentences $p_1, p_2, \dots, p_N$. For example, for three sentences p_1, p_2 , and p_3 , we have a graph of the distances shown in Fig. 3. This graph can be represented by a distance matrix

$$D = \begin{bmatrix} 0 & D_{1,2} & D_{1,3} \\ D_{1,2} & 0 & D_{2,3} \\ D_{1,3} & D_{2,3} & 0 \end{bmatrix},$$

which has a general form of a symmetric matrix,

$$D = \begin{bmatrix} 0 & D_{1,2} & \dots & D_{1,N} \\ D_{1,2} & 0 & \dots & D_{2,N} \\ \dots & \dots & \dots & \dots \\ D_{1,N} & D_{2,N} & \dots & 0 \end{bmatrix}$$

Having distances D_{ij} between propositions we can use them to compute complex expressions in agents' logic operation. For instance, using $0 \leq D(p, q) \leq G(p) + G(q)$ and $G(p \wedge q) \equiv G(p \vee q) - G((\neg p \wedge q) \vee (p \wedge \neg q)) \equiv G(p \vee q) - D(p, q)$

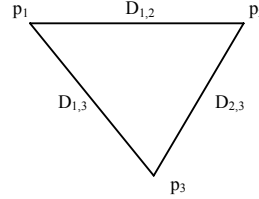


Fig. 3. Graph of the distances for sentences p_1, p_2, p_3

V. NEURAL IMAGE OF AUT

In this section, we show the possibility for a new type of neural network based on the AUT. This type of the neural network is dedicated to computation of many-valued logic operations used to model uncertainty process. The traditional neural networks model Boolean operations and classical logic. In a new fuzzy neural network, we combine two logic levels (classical and fuzzy) in the same neural network as presented in Fig. 4.

Figs 5-7 at the end of this paper show that at the first level we have the ordinary Boolean operations (NOT, OR, AND). At the second level, the network fuses results of different Boolean operations. As a result, a many value logic value is generated as Fig. 4 shows.

Next, we present an example of many-valued logic operation AND with agents and fusion in the neural network. Table 1 shows the individual AND operation in the population of two agents.

Table 1. Agent Boolean logical rule for $p \wedge q$

$p \backslash q$	$\begin{pmatrix} na_1 & na_2 \\ T & T \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ T & F \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ F & T \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ F & F \end{pmatrix}$
$\begin{pmatrix} na_1 & na_2 \\ T & T \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ T & T \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ T & F \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ F & T \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ F & F \end{pmatrix}$
$\begin{pmatrix} na_1 & na_2 \\ T & F \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ T & F \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ T & F \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ F & F \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ F & F \end{pmatrix}$
$\begin{pmatrix} na_1 & na_2 \\ F & T \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ F & T \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ F & F \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ F & F \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ F & F \end{pmatrix}$
$\begin{pmatrix} na_1 & na_2 \\ F & F \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ F & F \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ F & F \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ F & F \end{pmatrix}$	$\begin{pmatrix} na_1 & na_2 \\ F & F \end{pmatrix}$

With the fusion process in AUT, we have the many-valued logic in the neural network. In table 2, we use the aggregation rule by which we can generate a many-valued logic structure with the following three logic values:

$$\Omega = \left\{ true, \frac{true - false}{2}, false \right\} = \left\{ T, \frac{T - F}{2}, F \right\}$$

with equivalent notations

$$\frac{T + F}{2} = \frac{F + T}{2} = \frac{T}{2} = \frac{F}{2} = \frac{1}{2}T = \frac{1}{2}F$$

We can write the previous composition rule in the simple form shown in Table 3 where different results for the same couple of elements are located in the same cell.

Table 3 contains two different results for the AND operation,

$$p \wedge q = \frac{\text{true}}{2} \wedge \frac{\text{true}}{2} = \left\{ \text{false}, \frac{\text{true}}{2} \right\}$$

for $p = \frac{1}{2}$ true and $q = \frac{1}{2}$ true. In this case, we have no criteria to choose one or the other. Here the operation AND is not defined, we have two possible results one is false and the other is $\frac{1}{2}$ true.

Table 2. Neuronal fusion process for $p \wedge q$

$p \wedge q$	$\frac{T+T}{2} = T$	$\frac{T+F}{2} = \frac{T}{2}$	$\frac{F+T}{2} = \frac{T}{2}$	$\frac{F+F}{2} = F$
$\frac{T+T}{2} = T$	$\frac{T+T}{2} = T$	$\frac{T+F}{2} = \frac{T}{2}$	$\frac{F+T}{2} = \frac{T}{2}$	$\frac{F+F}{2} = F$
$\frac{T+F}{2} = \frac{T}{2}$	$\frac{T+F}{2} = \frac{T}{2}$	$\frac{T+F}{2} = \frac{T}{2}$	$\frac{F+F}{2} = F$	$\frac{F+F}{2} = F$
$\frac{F+T}{2} = \frac{T}{2}$	$\frac{F+T}{2} = \frac{T}{2}$	$\frac{F+F}{2} = F$	$\frac{F+T}{2} = \frac{T}{2}$	$\frac{F+F}{2} = F$
$\frac{F+F}{2} = F$	$\frac{F+F}{2} = F$	$\frac{F+F}{2} = F$	$\frac{F+F}{2} = F$	$\frac{F+F}{2} = F$

Now, having the commutative rule we derive,

$$\frac{T+F}{2} = \frac{F+T}{2} = \frac{T}{2}$$

Table 3 Simplification of Table 2

$p \wedge q$	$\frac{T+T}{2} = T$	$\frac{T}{2}$	$\frac{F+F}{2} = F$
$\frac{T+T}{2} = T$	$\frac{T+T}{2} = T$	$\frac{T+T}{2} = T$	$\frac{F+F}{2} = F$
$\frac{T}{2}$	$\frac{F+T}{2} = \frac{T}{2}$	$\frac{F+F}{2} = F$ $\frac{F+T}{2} = \frac{T}{2}$	$\frac{F+F}{2} = F$
$\frac{F+F}{2} = F$	$\frac{F+F}{2} = F$	$\frac{F+F}{2} = F$	$\frac{F+F}{2} = F$

Table 4 First operation

$p \wedge q$	$\frac{T+T}{2} = T$	$\frac{T}{2}$	$\frac{F+F}{2} = F$
$\frac{T+T}{2} = T$	$\frac{T+T}{2} = T$	$\frac{T+T}{2} = T$	$\frac{F+F}{2} = F$
$\frac{T}{2}$	$\frac{F+T}{2} = \frac{T}{2}$	$\frac{F+F}{2} = F$	$\frac{F+F}{2} = F$
$\frac{F+F}{2} = F$	$\frac{F+F}{2} = F$	$\frac{F+F}{2} = F$	$\frac{F+F}{2} = F$

Table 5. Second operation

$p \wedge q$	$\frac{T+T}{2} = T$	$\frac{T}{2}$	$\frac{F+F}{2} = F$
$\frac{T+T}{2} = T$	$\frac{T+T}{2} = T$	$\frac{T+T}{2} = T$	$\frac{F+F}{2} = F$
$\frac{T}{2}$	$\frac{F+T}{2} = \frac{T}{2}$	$\frac{T}{2}$	$\frac{F+F}{2} = F$
$\frac{F+F}{2} = F$	$\frac{F+F}{2} = F$	$\frac{F+F}{2} = F$	$\frac{F+F}{2} = F$

The first one is shown in Table 4 and the second one is shown in Table 5.

VI. EXAMPLE OF APPLICATION

Consider an agent g_1 (radar 1) that evaluates statements p and q : (p) -- a set of radar measurements m_{11} belongs to track r_{11} produced by object a , and (q) -- a set of radar measurements m_{12} belongs to track r_2 produced by object

b . The allowed binary answers are T or F for p and q statements. Similarly, another agent g_2 (infrared sensor, IR) evaluates (p) -- if a set of its measurements m_{21} belongs to its track r_{21} produced by object a , and (q) -- if a set of its measurements m_{22} belongs to its track r_{22} produced by object b . The answers can contradict to each other. We may get $v_1(p)=T$, $v_1(q)=F$ and $v_2(p)=F$, $v_2(q)=F$. In our terms $v_1(p \wedge q)=(TF)$, $v_2(p \wedge q)=(TF)$. For both agents we have a vector of evaluations $\mathbf{v}(p \wedge q) = (T \wedge T, F \wedge T) = (T, T/2)$. It can be fused further to a single truth value, $\mu(p \wedge q) = (T, T/2) = T$ as defined in Table 3. The last single T value for $p \wedge q$ follows the majority voting principle that is common in neural network modeling. We argue that vector reasoning should be continued as long as possible to preserve the richness of the information. This is especially critical having more than two contradictory agents and more complex expressions with several AND, OR and NOT operations combined.

This illustrative example is a small fraction of a potentially very large area of applications of agent-based neural networks for sensor analysis and fusion. Among specific tasks are tracking and discrimination objects in medical imaging and national security domains.

Uncertainty is a fundamental issue in sensor fusion. Measurements are not exact. Their uncertainties are multidimensional, non-static and differ from sensor to sensor. They depend on sensor resolution, locations of sensors and multiple objects, viewing geometry, changing environment and other factors. A variety of models has been built for sensor fusion. They use the probability theory, fuzzy sets and logic, Demster-Shaffer evidence theory, stochastic differential equations, Bayesian models, Kalman filters and other models.

Neural networks have been used to provide better adaptation of Kalman filters as well as standalone models for a situation with highly non-linear motions.

Fuzzy logic was used to build "fuzzy" sensors to overcome the intrinsic difficulty to control the uncertainty in the sensor analysis. In this way, fuzzy neurons give a logic component to inconsistent or conflicting description of the sensors such radars or infrared sensors.

Currently there are two main difficulties in the fuzzy neuron concept for sensors. First, we must assign a membership distribution to the fuzzy sensor. In many cases, this is difficult. We may end up with many different competing distributions. The fuzzy distribution depends not only on the sensor, but also on how the sensor is perceived by the observer that creates the fuzzy distribution. Second, when we model a sensor as a fuzzy sensor with fuzzy neurons we embed fuzzy logic operations into the neurons. This itself presents many difficulties due to the impossibility to define in advance the fuzzy logic operations as AND, OR and NOT that will be suitable to the task at hand. In fuzzy logic, these operations are connected with a concrete situation of sensor measurements. We must select one AND operation from many t-norms and one OR operation from many t-c-norms. Here we lose the uniqueness of the logic operations in contrast with the unique classical logic operations.

By introducing agent neuron uncertainty, we solve these two problems. For the first one, we consider any sensor as an agent that can provide a True or False value for a given sensor measurement. Many sensors give an n-tuple of true and false values as representation of conflicting measurements among sensors or agents. Now, having n-tuples of the true and false for n sensors, we can define one and only one many-valued logic operation. For a particular measurement situation, we define the fundamental logic operations to obtain logic conclusions about measurements provided by n sensors. In this way, the uncertainty is connected only to the conflicting measurements not to the observer. This reduces the complexity of a fuzzy set identification in neuro-fuzzy models. Next, to build neuro-fuzzy models for tracking, we need to identify a type of logic operations with fuzzy sets. Fuzzy logic offers multiple logic operations (t-norms and t-co-norms). The selection of them is very non-trivial. The advantages of the agents approach here is that it does not involve an external entity to decide what type of logic operations is necessary to use. In summary, the new contribution of the agent approach to the traditional fuzzy-neural network learning is in simplification of the fuzzy neuron learning process by the direct use of measurements of the sensors (data) with a fixed and “self-defined” logic for obtaining conclusions.

VII. CONCLUSION

This paper presented the Agent-based Uncertainty Theory (AUT) for Neural Networks. AUT can model the inconsistent logic values. Now with the AUT, it is possible to generate neuron models and fuzzy neuron models that deal with intrinsic conflicting situations and inconsistency. The AUT opens the opportunities to rebuild previous models of uncertainty with the explicit and consistent introduction of conflicting and inconsistent phenomena by the means of the many-valued logic.

REFERENCES

- [1] R. Carnap, R. Jeffrey, *Studies in Inductive Logics and Probability*, vol. 1, 35-165 Berkeley, CA, University of California Press, 1971
- [2] R. Fagin J. Halpern J. “Reasoning about knowledge and Probability”, *Journal of the ACM*, 41, 2 340 – 367, 1994.
- [3] B. Edmonds, “Review of reasoning about rational agents by Michael Wooldridge”, vol. 5. Issue 1, *Journal of Artificial Societies and Social Simulation*, Available: <http://jasss.soc.surrey.ac.uk/5/1/reviews/edmonds.html>, 2002
- [4] J. Ferber, *Multi agent systems*, Addison Wesley, 1999
- [5] G. Gigerenzer, R., Selten, *Bounded rationality*, Cambridge: The MIT Press, 2002
- [6] J. Halpern J. *Reasoning about uncertainty*, MIT Press, 2005
- [7] E. Hisdal E. *Logical structures for representation of knowledge and uncertainty*, Springer, 1998
- [8] G. Resconi, L. Jain, *Intelligent agents*, Springer Verlag, 2004
- [9] G. Resconi, G. Klir G., U. St. Clair, “Hierarchical uncertainty metatheory based upon modal logic”. *International Journal of General Systems* 21, 23-50, 1992
- [10] G. Resconi, G.Klir, U. St. Clair, and D.Harmanec, “The integration of uncertainty theories”. *Intern. J. Uncertainty Fuzziness knowledge-Based Systems* 1, 1-18, 1993.
- [11] G. Resconi, G., Klir, D. Harmanec, , U. St. Clair, “Interpretation of various uncertainty theories using models of modal logic: a summary”, *Fuzzy Sets and Systems* 80 (1996) 7-14
- [12] D. Harmanec, G. Resconi, G., Klir, and Yin Pan, “On the computation of uncertainty measure in Dempster-Shafer theory, *International Journal General System*”, vol.25 (2) pp.153-163, 1995
- [13] G. Resconi, T. Murai, M. Shimbo, “Field theory and modal logic by semantic field to make uncertainty emerge from information”, *Int. Journal General System*, vol. 29 (5) pp.737-782, 2000
- [14] G. Resconi I. Turksen, “Canonical forms of fuzzy truthhoods by Meta-Theory Based Upon Modal Logic”, *Information Sciences* 131, 157-194, Elsevier, 2001
- [15] G. Resconi B. Kovalerchuk, “The logic of uncertainty with irrational agents”, In: *Proc. of JCIS-2006, Advances in Intelligent Systems Research*, Taiwan, Atlantis Press, 2006.
- [16] D. Kahneman, “Maps of Bounded Rationality: Psychology for Behavioral Economics”. *The American Economic Review*. 93(5), pp. 1449-1475, 2003
- [17] B. Kovalerchuk “Analysis of Gaines’ logic of uncertainty”, In: *Proceeding of NAFIPS ’90* vol.2 edited by I.B. Turksen, Toronto, Canada, pp.293-295, 1990
- [18] B. Kovalerchuk, “Context spaces as necessary frames for correct approximate reasoning”. *International Journal of General Systems*, v.25, n 1, 61-80, 1996.
- [19] B. Kovalerchuk, E. Vityaev, *Data mining in finance: advances in relational and hybrid methods*, Kluwer, 2000
- [20] M. Wooldridge, *Reasoning about Rational Agents*, Cambridge, MA, The MIT Press, 2000
- [21] J. Montero, D. Gomez, H. Bustine, “On the relevance of some families of fuzzy sets”, *Fuzzy Sets and Systems*, 16: 2429-2442, 2007
- [22] K.T. Atanassov. *Intuitionistic Fuzzy Sets*, Physica Verlag, Springer, Heidelberg, 1999
- [23] C. Flament, *Applications of graphs theory to group structure*, Prentice Hall , London, 1963
- [24] E.H. Ruspini “A new approach to clustering”. *Information and Control*, 15:22-32, 1999.
- [25] G. Priest, K. Tanaka, *Paraconsistent Logic*. Stanford Encyclopedia of Philosophy. Available: <http://plato.stanford.edu/entries/logic-paraconsistent>, 2004.
- [26] G. Chalkiadakis, C. Boutilier, “Sequential Decision Making in Repeated Coalition Formation under Uncertainty”, In *Proc. of 7th Int. Conf. on Autonomous Agents and Multiagent Systems (AAMAS 2008)*, Padgham, Parkes, Müller and Parsons (eds.), May, 12-16, 2008, Estoril, Portugal, <http://eprints.ecs.soton.ac.uk/15174/1/BayesRLCF08.pdf>
- [27] Von-Wun Soo, Agent negotiation under uncertainty and risk, In: *Design and applications of intelligent agents, Lecture Notes in Computer Science*, Vol. 1881, 2000, 31-45, Springer Berlin, R. Sun R., D. Qi, Rationality assumptions and optimality of co-learning, In *Design and Applications of Intelligent Agents, Lecture Notes in Computer Science*, Vol. 1881, 61-75, 2000, Springer
- [28] B. Baki, M. Bouzid, A.Ligeza, A Mouaddib, “A centralized planning technique with temporal constraints and uncertainty for multi-agent systems”, *Journal of Experimental & Theoretical Artificial Intelligence*, vol. 18, Number 3, 2006 , pp. 331-364(34)
- [29] C. van Dintner, *Adaptive bidding in single-sided auctions under uncertainty: An Agent-based Approach in Market Engineering* (Whitestein Series in Software Agent Technologies and Autonomic Computing). 2007, Birkhäuser, Basel
- [30] Wei Wu, E. Ekaette, B. Homayoun Far, “Uncertainty management framework for multi-agent system”, *Proceedings of ATS 2003*, 2003, Available: <http://www.enel.ucalgary.ca/People/far/pub/papers/2003/ATS2003-06.pdf>
- [31] M. Colyvan, “The philosophical significance of Cox's theorem”, *International Journal of Approximate Reasoning*, Vol. 37, No. 1, pp. 71–85, 2004.
- [32] M. Colyvan, “Is Probability the Only Coherent Approach to Uncertainty?” *Risk Analysis*, 28: 645–652, 2008
- [33] G. Resconi, B. Kovalerchuk, “Fusion in agent-based uncertainty theory and neural image of uncertainty”, In: *The 2008 IEEE World Congress on Computational Intelligence (WCCI 2008)*, Hong Kong, IEEE 2008, 3537-3543.

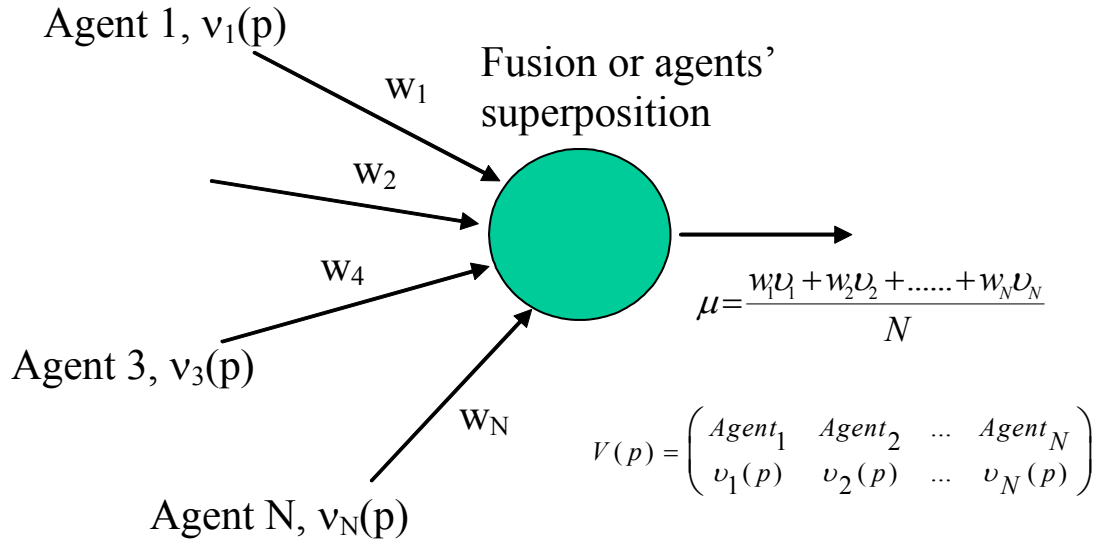


Fig. 4. Many-valued logic in the AUT neural network

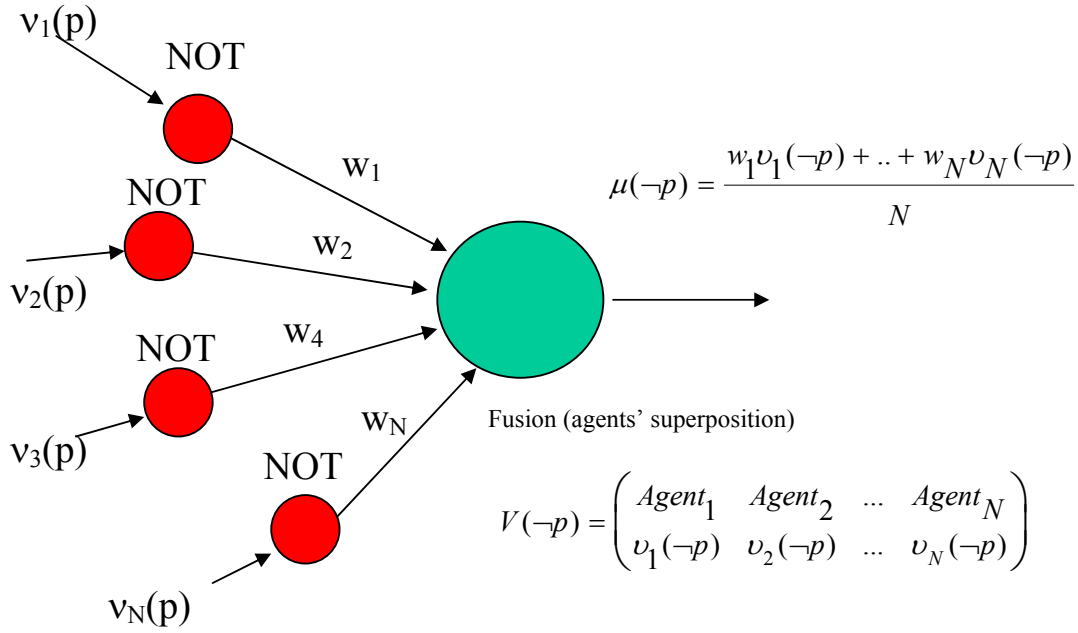


Fig. 5 Many-valued logic NOT operation in the AUT neural network

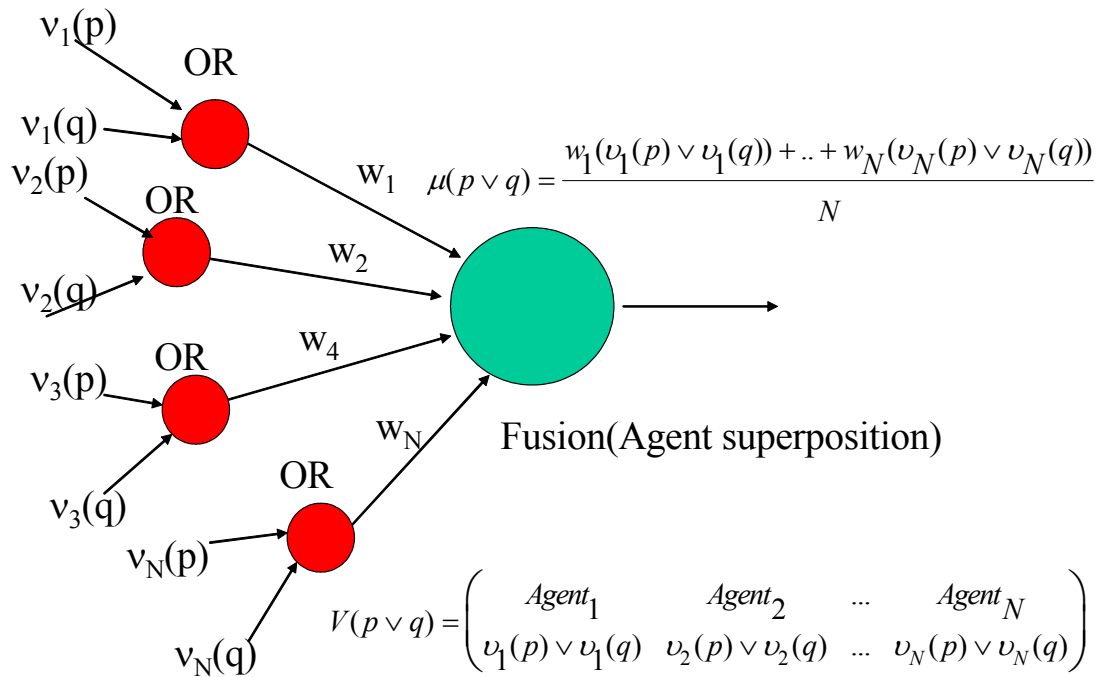


Fig. 6. AUT: Many-valued logic operation OR in the AUT neural network

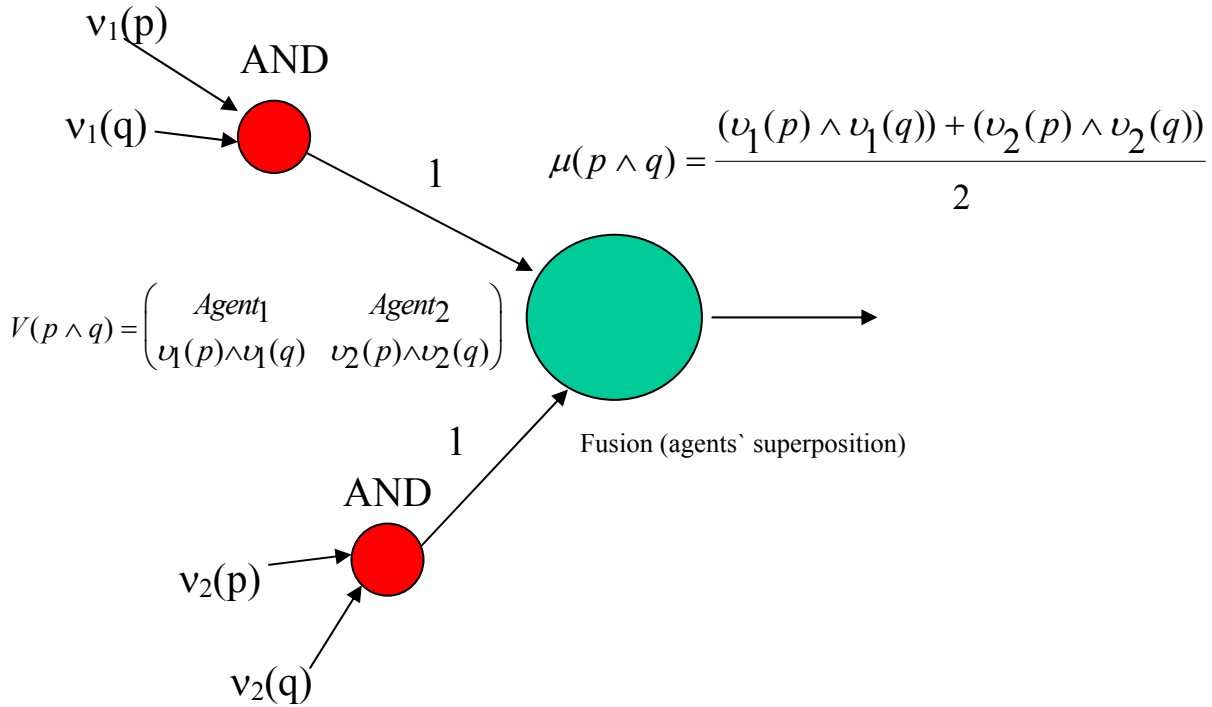


Fig. 7. Many-valued logic operation AND in the AUT neural network.