

Agent-based Uncertainty Logic Network

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Abstract—Boolean and discrete networks play an important role in many domains such as cellular automata. This paper generalizes that concept of Boolean networks for complex situations with multiple agents acting under uncertainty. This paper creates a logic network using a concept of the Agent-based Uncertainty Theory (AUT). The AUT is based on complex fusion of crisp (non-fuzzy) conflicting judgments of agents. It provides a uniform representation and an operational empirical interpretation for several uncertainty theories such as rough set theory, fuzzy sets theory, evidence theory, and probability theory. The AUT models conflicting evaluations that are fused in the same evaluation context. An AUT network extend the traditional inferential process by using a set of logic matrices obtained from AUT logic evaluation samples connected in a network. This network computes transformations of AUT logic vectors and gives logic rules for uncertainty situation. The AUT logic network is a generalization of the Boolean network. A Boolean network consists of a set of Boolean variables whose states are determined by other variables in the network. An AUT logic network consists of a set of agents presented as vector variables whose states or logic vector evaluations are determined by other variables in the network.

I. INTRODUCTION

Boolean networks are important in many domains such as the Internet, social networks, metabolic and protein networks, ecological networks, and genetic networks [Aldana, 2003]. We model logic network by Agent-based Uncertainty Theory (AUT) that uses complex fusion of crisp conflicting judgments of agents.

There are multiple challenges in modeling agents under uncertainty [1-20],[22-29], AUT represents and interprets uniformly several uncertainty theories such as rough set theory, fuzzy sets theory, evidence theory, and probability theory. AUT exploits the fact that agents as independent entities can give conflicting evaluations of the same attribute. It models conflicting evaluations that are fused in the same evaluation context. If only one evaluation is allowed for each statement in each context (world) as in the modal logic then there is no logical uncertainty. The situation that the AUT models is inconsistent (fuzzy) and is very far from the situation that modeled by the traditional logic that assumes consistency. We argue that the AUT by incorporating such inconsistent statements is able to model

different types of conflicts and their fusion known in many-valued logics, fuzzy logic, probability theory and other theories.

In the AUT approach, network of transformations Z are interpreted as logic operations in logic computations. The AUT logic network is based on the use of the logic of the uncertainty instead of the classical logic. The motivation for such AUT logic network is to provide high flexibility and logic adaptation that is useful form many domains including brain modeling, where communication among agents can be specified by the fusion process in the neural elaboration.

The probability calculus does not incorporate explicitly the concepts of irrationality or logic conflict of agent's state. It misses structural information at the level of individual objects, but preserves global information at the level of a set of objects. Given a dice the probability theory studies frequencies of the different faces $E=\{e\}$ as independent (elementary) events. This set of elementary events E has *no structure*. It is only required that elements of E are *mutually exclusive* and *complete*, that is no other alternative is possible. The order of its elements is irrelevant to probabilities of each element of E . No irrationality or conflict is allowed in this definition relative to mutual exclusion. The classical probability calculus does not provide a mechanism for modeling uncertainty when agents communicate (collaborates or conflict) in violation of mutual exclusion and completeness.. Recent work by Halpern [6] is an important attempt to fill this gap.

This paper is organized as follows: Sections 2 and 3 provide a summary of the AUT starting from concepts and definitions. Section 4 presents the AUT logic network for many-valued logic computation. We built the network operators Z by using matrices of samples that represent agents. Given the network as a Boolean network, we define AUT logic variables which values are defined by the other AUT logic values in the network.

II. CONCEPTS AND DEFINITIONS

Now we will define AUT concepts more formally first for individual agents and then for sets of agents. Consider a set of agents $G=\{g_1, g_2, \dots, g_n\}$. Each agent g_k assigns binary true/false value $v \in \{\text{True}, \text{false}\}$ to proposition p . To show that v was assigned by the agent g_k we use notation $g_k(p) = v_k$.

Definition. A triple $g = \langle N, A_R, A_A \rangle$ is called an *agent g* if N is label that is interpreted as agent's name and A_R is as set of truth-evaluation actions and A_A is a set of non- truth-evaluation actions associated with name N .

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For instance, agent g called Professor has a set of truth-evaluation actions A_R such as grading students' answer, while delivering a lecture is in another category A_A .

Definition. An agent g is called a *reasoning agent* if g assigns a truth-value $v(p)$ to any proposition p from a set of propositions S and any logical formula based on S .

The actions of a general agent may or may not include truth-evaluation actions. From a mathematical viewpoint a reasoning agent g serves as a mapping,

$$g: p \rightarrow v(p)$$

While natural agents have both AR , and AA , artificial software agents and robots may have only one of them.

Definition. A set of reasoning agents G is called *totally consistent* for proposition p if any agent g from $\{g\}$ always provides the same truth value for p .

In other words, all agents in G are in concord or logical coherence for the same proposition. Thus, changing the agent does not change logic value $v(p)$ for a totally consistent set of agents. Here $v(p)$ is *global* for G and has *no local variability* that is independent on the individual agent's evaluation. The classical logic is applicable for such set of consistent (rational) agents.

Definition. A set of reasoning agents G is called *inconsistent* for proposition p if there are two subset of agents G_1, G_2 such that agents from them provides different truth values for p .

Multiple reasons lead to agents' inconsistency. The most general one is the context (or hidden variables) in which each agent is evaluating statement p . Even in a relatively well formalized environment, context is not fully defined. Agent g_1 may evaluate p in context "abc", but agent g_2 in context "ab". Agent (robot) g_2 may have no sensor to obtain signal c or abilities to process and reason correctly with input from that sensor (e.g., color blind agent with a panchromatic camera).

Definition. Let S be a set propositions, $S=\{p_1, p_2, \dots, p_n\}$ then set

$$\neg S = \{\neg p_1, \neg p_2, \dots, \neg p_n\}$$

is called a *complementary set* of S .

Definition. A set of reasoning agents G is called *S-only-consistent* if agents $\{g\}$ are consistent only for propositions in $S=\{p_1, p_2, \dots, p_n\}$ and are inconsistent in the complimentary set $\neg S$.

The evaluations of p is a vector-function

$$v(p)=(v_1(p), v_2(p), \dots, v_n(p))$$

for a set of agents G that we will represent as follows:

$$v(p) = \begin{pmatrix} g_1 & g_2 & \dots & g_{n-1} & g_n \\ v_1 & v_2 & \dots & v_{n-1} & v_n \end{pmatrix} \quad (1)$$

An example of the logic evaluation by five agent is shown below for $p = "A > B"$

$$f(p) = \begin{pmatrix} g_1 & g_2 & g_3 & g_4 & g_5 \\ true & false & true & false & true \end{pmatrix}$$

Here $A > B$ is true for agents g_1, g_3 , and g_5 and it is false for the agents g_2 , and g_4 .

Kolmogorov's axioms of the probability theory are based on a totally consistent set of agents (for a set of statements) on mutual exclusion of elementary events. It follows from the definitions below for a set of events $E=\{e_1, e_2, \dots, e_n\}$.

Definition. Set $E=\{e_1, e_2, \dots, e_n\}$ is called a set of *elementary events* (or mutually exclusive events) for predicate El if

$$\forall e_i, e_j \in A \quad El(e_i) \vee El(e_j) = \text{True} \quad \text{and} \quad El(e_i) \wedge El(e_j) = \text{False}.$$

In other words, event e_i is an *elementary event* ($El(e_i)=\text{true}$) if for any $j, j \neq i$ events e_i and e_j cannot happen simultaneously, that is probability $P(e_i \wedge e_j) = 0$ and $P(e_i \vee e_j) = P(e_i) + P(e_j)$. Property $P(e_i \wedge e_j) = 0$ is the *mutual exclusion axiom* (ME- axiom).

Let $S=\{p_1, p_2, \dots, p_n\}$ be a set of statements, where $p_i=p(e_i)=\text{True}$ if and only if event e_i is an elementary event. In probability theory, $p(e_i)$ is not associated with **any** specific agent. It is assumed a global property (applicable to all agents). In other words, statements $p(e_i)$ are totally consistent for all agents.

Definition. A set of agent $S = \{g_1, g_2, \dots, g_n\}$ is called a *ME-rational* set of agents if

$$\forall e_i, e_j \in A, \forall g_i \in S, \quad p(e_i) \vee p(e_j) = T \quad \& \quad (e_i \wedge e_j) = F.$$

In other words, these agents are *totally consistent* or *rational on Mutual Exclusion*. Previously we assumed that each agent assigns value $v(p)$ and we did not model this process explicitly. Now we introduce a set of criteria $C=\{C_1, C_2, \dots, C_m\}$ by which an agent can decide if a proposition p is true or false, i.e., now $v(p) = v(p, C_i)$, which means that p is evaluated by using the criterion C_i .

Definition Given a set of criteria C , agent g is in a *self-conflicting state* if

$$\exists C_i, C_j (C_i, C_j \in C) \quad \& \quad v(p, C_i) \neq v(p, C_j)$$

In other words, an agent is in a self-conflicting state if two criteria exist such that p is true for one of them and false for another one. With the explicit set of criteria, the logic

evaluation function $\mathbf{v}(p)$ is not vector-function any more, but it is expanded to be a matrix function as shown below:

$$\mathbf{v}(p) = \begin{bmatrix} & g_1 & g_2 & \dots & g_n \\ C_1 & v_{1,1} & v_{1,2} & \dots & v_{1,n} \\ C_2 & v_{2,1} & v_{2,2} & \dots & v_{2,n} \\ \dots & \dots & \dots & \dots & \dots \\ C_m & v_{m,1} & v_{m,2} & \dots & v_{m,n} \end{bmatrix} \quad (2)$$

For example, four agents using four criteria can produce $\mathbf{v}(p)$ as follows:

$$\mathbf{v}(p) = \begin{bmatrix} & g_1 & g_2 & g_3 & g_4 \\ C_1 & true & false & false & true \\ C_2 & false & false & true & true \\ C_3 & true & false & true & true \\ C_4 & false & false & true & true \end{bmatrix}$$

Note that introduction of a set of criteria $\mathbf{C}$ explains self-conflict, but does not remove it. The agent still needs to resolve the ultimate preference contradiction having a goal, say, to buy only one and better car. It also does not resolve conflict among different agents if they need to buy a car jointly.

If agent g can modify criteria in $\mathbf{C}$ making them consistent for p then agent g can resolves self-conflict. The agent can be in a logic conflict state because of inability to understand the complex *context* and to evaluate *criteria*. For example, in the stock market environment, some traders quite often do not understand a complex context and rational criteria of trading. These traders can be in logic conflict exhibiting chaotic, random, and impulsive behavior. They can sell and buy stocks, exhibiting logic conflicting states “sell” = p and “buy” = $\neg p$ in the same market situation that appears as irrational behavior, which means that both p and $\neg p$ can be true and the statement $p \wedge \neg p$ can be true. In contrast, the rational agent does not mix sell or buy signals in the fixed market situation that is $p \wedge \neg p$ is false.

A logical structure of self-conflicting states and agents is much more complex than it is without self-conflict. For m binary criteria $C_1, C_2, \dots, C_m$ that evaluate the logic value for the same attribute, there are 2^m possible states and only two of them (all true or all false values) do not exhibit conflict between criteria.

III. FRAMEWORK OF FIRST ORDER OF CONFLICT LOGIC STATE

A. Definitions

Definition. A set of agents G is in a first order of conflicting logic state (*first order conflict*, for short) if

$$\exists g_i, g_j \ (g_i, g_j \in G) \ \& \ \mathbf{v}(p, g_i) \neq \mathbf{v}(p, g_j).$$

In other words, there are agents g_i and g_j in G for which exist different values v_i, v_j in

$$\mathbf{v}(p) = \begin{pmatrix} g_1 & g_2 & \dots & g_{n-1} & g_n \\ v_1 & v_2 & \dots & v_{n-1} & v_n \end{pmatrix}$$

A set of agents G is in the *first order of conflict* if

$$G(A > B) \cap G(A < B) = \emptyset, \ G(A > B) \neq \emptyset, \ G(A < B) \neq \emptyset$$

$$G(A > B) \cup G(A < B) = G.$$

The following definition presents this idea in general terms.

Definition. A set of agents G is in a **First Order Conflict (FOC)** for proposition p if

$$G(p) \cap G(\neg p) = \emptyset, \text{ and } G(p) \neq \emptyset, \ G(\neg p) \neq \emptyset,$$

$$G(p) \cup G(\neg p) = G.$$

Fig. 1 shows a set of 20 agents in the logic conflicting state, where 7 white agents are in the state True and 13 black agents are in the state False for the same proposition $p = "A > B"$.

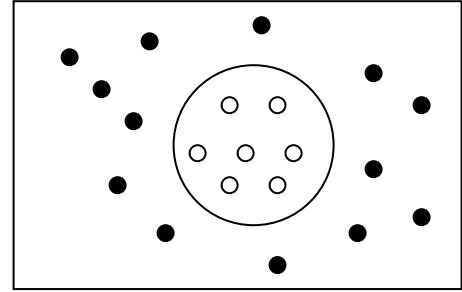


Fig. 1. A set of 20 agents in the first order of logic conflict

Below we show that at the first order of conflicts, AND and OR operations should differ from the classical logic operations and should be **vector operations** in the space of the agents' evaluations (*agents space*). The vector operations reflect a structure of logic conflict among coherent individual agent evaluations.

B. Fusion process

If a single decision must be made at the first order of conflict, then we must introduce a **fusion process** of the logic values of proposition p given by all agents. A basic way to do this is to compute the weighted frequency of logic value given by all agents:

$$\mu(p) = w_1 v_1(p) + \dots + w_n v_n(p) = \begin{bmatrix} v_1(p) \\ v_2(p) \\ \dots \\ v_n(p) \end{bmatrix}^T \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{bmatrix} \quad (3)$$

where $\begin{bmatrix} v_1(p) \\ v_2(p) \\ \dots \\ v_n(p) \end{bmatrix}$ and $\begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{bmatrix}$ are two vectors in the space of the

agents' evaluations. The first vector contains all logic states (True/False) for all agents, the second vector (with property $\sum_{k=1}^n w_k = 1$) contains non-negative weights (utilities) that are

given to each agent in the fusion process. In a simple frequency case, each weight is equal to $1/n$. At first glance, $\mu(p)$ is the same as used in the probability and utility theories. However, classical axioms of the probability theory have no references to agents producing initial uncertainty values and do not violate the mutual exclusion. Below we define **vector logic operations** for the first order of conflict logic states $\mathbf{v}(p)$.

Definition

$$\mathbf{v}(p \wedge q) = v_1(p) \wedge v_1(q), \dots, v_n(p) \wedge v_n(q),$$

$$\mathbf{v}(p \vee q) = v_1(p) \vee v_1(q), \dots, v_n(p) \vee v_n(q)$$

$$\mathbf{v}(\neg p) = \neg v_1(p), \dots, \neg v_n(p),$$

where the symbols $\wedge, \vee, \neg$ in the right side of the equations are the classical AND, OR, and NOT operations.

Below these operations are written with explicit indication of agents (in the first row):

$$\mathbf{v}(p \wedge q) = \begin{pmatrix} g_1 & g_2 & \dots & g_{n-1} & g_n \\ v_1(p) \wedge v_1(q) & v_2(p) \wedge v_2(q) & \dots & v_{n-1}(p) \wedge v_{n-1}(q) & v_n(p) \wedge v_n(q) \end{pmatrix}$$

$$\mathbf{v}(p \vee q) = \begin{pmatrix} g_1 & g_2 & \dots & g_{n-1} & g_n \\ v_1(p) \vee v_1(q) & v_2(p) \vee v_2(q) & \dots & v_{n-1}(p) \vee v_{n-1}(q) & v_n(p) \vee v_n(q) \end{pmatrix}$$

$$\mathbf{v}(\neg p) = \begin{pmatrix} g_1 & g_2 & \dots & g_{n-1} & g_n \\ v_1(\neg p) & v_2(\neg p) & \dots & v_{n-1}(\neg p) & v_n(\neg p) \end{pmatrix}$$

Below we present the important properties of sets of conflicting agents at the first order of conflicts. Let $|G(x)|$ be the numbers of agents for which proposition x is true. Statement 1 sets up properties of the AND and OR operations for nested sets of conflicting agents.

Statement 1 (general non min/max properties of $\wedge$ and $\vee$ operations)

If G is a set of agents at the first order of conflicts and

$$|G(q)| \leq |G(p)|$$

then $\wedge$ and $\vee$ logic operations satisfy the following properties

$$|G(p \wedge q)| = \min(|G(p)|, |G(q)|) - |G(\neg p \wedge q)|, \quad (4)$$

$$|G(p \vee q)| = \max(|G(p)|, |G(q)|) + |G(\neg p \wedge q)| \quad (5)$$

If G is a set of agents at the first order of conflicts and

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then $\wedge$ and $\vee$ logic operations satisfy the following properties

$$|G(p \wedge q)| = \min(|G(p)|, |G(q)|) - |G(\neg q \wedge p)|,$$

$$|G(p \vee q)| = \max(|G(p)|, |G(q)|) + |G(\neg q \wedge p)|.$$

and also

$$|G(p \vee q)| = |G(p) \cup G(q)|, |G(p \wedge q)| = |G(p) \cap G(q)|$$

Corollary 1 (min/max properties of $\wedge$ and $\vee$ operations for nested sets of agents)

If G is a set of agents at the first order of conflicts such that $G(q) \subset G(p)$ or $G(p) \subset G(q)$ then

$$G(\neg p \wedge q) = \emptyset \text{ or } G(\neg q \wedge p) = \emptyset$$

$$|G(p \wedge q)| = \min(|G(p)|, |G(q)|)$$

$$|G(p \vee q)| = \max(|G(p)|, |G(q)|)$$

This follows from the statement 1. The corollary presents a well-known condition when the use of min, max operations has the clear justification.

Let $G^c(p)$ is a *complement* of $G(p)$ in G : $G^c(p) = G \setminus G(p)$, $G = G(p) \cup G^c(p)$.

Statement 2. $G = G(p) \cup G^c(p) = G(p) \cup G(\neg p)$.

Corollary 2. $G(\neg p) = G^c(p)$

It follows directly from Statement 2.

Statement 3. If G is a set of agents at the first order of conflicts then

$$G(p \vee \neg p) = G(p) \cup G(\neg p) = G(p) \cup G^c(p) = G$$

$$G(p \wedge \neg p) = G(p) \cap G(\neg p) = G(p) \cap G^c(p) = \emptyset$$

It follows from the definition of the first order of conflict and statement 2. In other words, $G(p \wedge \neg p) = \emptyset$ corresponds to the contradiction $p \wedge \neg p$, that is always false and $G(p \vee \neg p) = G$ corresponds to the tautology $p \vee \neg p$, that is always true in the first order conflict.

Let $G_1 \oplus G_2$ be a *symmetric difference* of sets of agents G_1 and G_2 ,

$$G_1 \oplus G_2 = (G_1 \cap G_2^c) \cup (G_1^c \cap G_2)$$

and let $p \oplus q$ be the *exclusive or* of propositions p and q ,

$$p \oplus q = (p \wedge \neg q) \vee (\neg p \wedge q).$$

Consider, a set of agents $G(p \oplus q)$. It consists of agents for which values of p and q differ from each other, that is

$$G(p \oplus q) = G((p \wedge \neg q) \vee (\neg p \wedge q)).$$

Below we use the number of agents in set $G(p \oplus q)$ to define a measure of difference between statements p and q and a measure of difference between sets of agents $G(p)$ and $G(q)$.

Definition. A measure of difference $D(p, q)$ between statements p and q and a measure of difference $D(G(p), G(q))$ between sets of agents $G(p)$ and $G(q)$ are defined as follows:

$$D(p, q) = D(G(p), G(q)) = |G(p) \oplus G(q)|$$

Statement 4. $D(p,q) = D(G(p),G(q))$ is a distance, i.e., it satisfies distance axioms

$$D(p,q) \geq 0$$

$$D(p,q) = D(q,p)$$

$$D(p,q) + D(q,h) \geq D(p,h).$$

This follows from the properties of the symmetric difference $\oplus$ [e.g., Flament 1963].

Fig. 2 illustrates a set of agents $G(p)$ for which p is true and a set of agents $G(q)$ for which q is true. In Fig. 2(a) the number of agents for which truth values of p and q are different, $(\neg p \wedge q) \vee (p \wedge \neg q)$, is equal to 2. These agents are represented by white squares.

Therefore the distance between $G(p)$ and $G(q)$ is 2. Fig. 2(b) shows other $G(p)$ and $G(q)$ sets with the number of the agents for which $\neg p \wedge q \vee (p \wedge \neg q)$ is true equal to 6 (agents shown as white squares and squares with the grid). Thus, the distance between the two sets is 6.

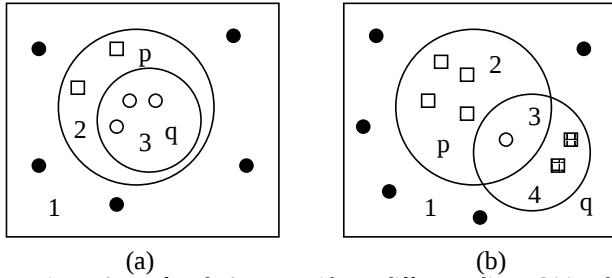


Fig. 2. A set of total 10 agents with two different splits to $G(p)$ and $G(q)$ subsets (a) and (b).

In Fig. 2(a), set 2 consists of 2 agents $|G((p \wedge \neg q))| = 2$ and set 4 is empty, $|G((\neg p \wedge q))| = 0$, thus $D(\text{Set2}, \text{Set4}) = 2$. This emptiness means that a set of agents with true p includes a set of agents with true q . In Fig. 2(b), set 2 consists of 4 agents $|G((p \wedge \neg q))| = 4$ and set 4 consists of 2 agents,

$$|G((\neg p \wedge q))| = 2, \text{ thus } D(\text{Set2}, \text{Set4}) = 6.$$

IV. AUT LOGIC NETWORK

A. AUT Logic Network Concepts

We define a *logic network* of input A and output B , where A and B are $m \times n$ matrices with $m \geq n$. In Fig. 3 we show one of the AUT logic networks.

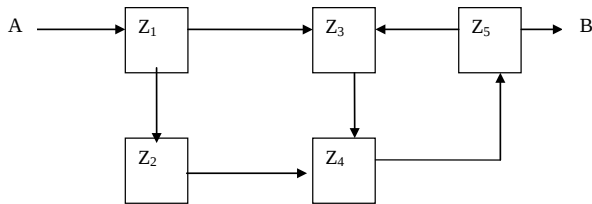


Fig. 3. An AUT logic network

Here

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2pn} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2pn} \\ \dots & \dots & \dots & \dots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{bmatrix} \quad (6)$$

The elements of A and B have values 1 (true) or 0 (false). Matrix Z_k is a logic transformation matrix. The matrixes A and B are matrices of logic evaluations of a given propositions p and q by the same set of agents. In a more formal notation

$$A = \begin{bmatrix} & S_1 & S_2 & \dots & S_n \\ g_1 & a_{11} & a_{12} & \dots & a_{1n} \\ g_2 & a_{21} & a_{22} & \dots & a_{2,n} \\ \dots & \dots & \dots & \dots & \dots \\ g_m & a_{m1} & a_{m2} & \dots & a_{m,n} \end{bmatrix}$$

where $g_1, g_2, \dots, g_m$ are m agents and $S_1, S_2, \dots, S_n$ are n samples of different logic evaluations of the same proposition in AUT, e.g.,

$$A(s) = \begin{bmatrix} & S_1 & S_2 \\ g_1 & 1 & 0 \\ g_2 & 1 & 0 \\ g_3 & 0 & 1 \\ g_4 & 0 & 1 \end{bmatrix}$$

In this example given the AUT logic proposition s , we have two logic evaluation samples S_1 and S_2 at the first level of AUT. These samples are two free logic evaluations given by four agents maybe in different circumstances, contexts (e.g., times or places). We remember that the maximum number of different samples for four agents is $2^4 = 16$.

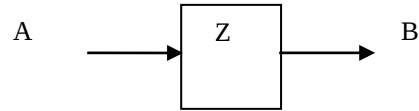


Fig. 4. Simple module in the AUT logic network

Expression (7) formally represents the input-output relation between matrices A and B in Fig. 4,

$$B = Z A \quad (7)$$

Using matrix algebra Z can be computed as shown in (8)

$$Z = B (A^T A)^{-1} A^T \quad (8)$$

because

$$Z A = B (A^T A)^{-1} A^T A = B$$

Proposition 1 :

If for given A and B $m > n$ then multiple different matrixes Z exist such that $B = Z A$.

Proof :

For $Z = Z + \Omega L^T$ we have $B = (Z + \Omega L^T) A = Z A + \Omega L^T A$. If $L^T A = 0$ and $L^T \neq 0$ then $B = Z A$. Thus, all such operators $Z + \Omega L^T$ transform A into B .

By solving the homogeneous equation $L^T A = 0$, or $(L^T A)^T = A^T L = 0$ given A we can find L and compute the family of the transformations $Z + \Omega L^T$ that transform A into B . For example, given two samples of evaluation by three agents as input A and two samples of evaluation as output B

$$A_1(p) = \begin{pmatrix} g_1 & g_2 & g_3 \\ 1 & 1 & 0 \end{pmatrix}$$

$$A_2(p) = \begin{pmatrix} g_1 & g_2 & g_3 \\ 0 & 1 & 1 \end{pmatrix}$$

$$B_1(p) = \begin{pmatrix} g_1 & g_2 & g_3 \\ 0 & 1 & 1 \end{pmatrix}$$

$$B_2(p) = \begin{pmatrix} g_1 & g_2 & g_3 \\ 1 & 1 & 0 \end{pmatrix}$$

we can get input A and output B matrixes of the AUT logic network, where A is formed from values of A_1 and A_2 and B is formed from values of B_1 and B_2 :

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

A_1 and A_2 are interpreted as answers of agents within two different contexts. B_1 and B_2 are interpreted similarly. The explicit contexts may not be known and contexts for A and B can differ. To solve the equation $A^T L = 0$ we can write:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} + \begin{bmatrix} 0 \\ L_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -L_3 \end{bmatrix}$$

Thus, the solution is

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -L_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -L_3 \end{bmatrix} = \begin{bmatrix} L_3 \\ -L_3 \end{bmatrix}$$

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} L_3 \\ -L_3 \\ L_3 \end{bmatrix}$$

and

$$Z = B(A^T A)^{-1} A^T + \Omega L^T =$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}^T + \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \begin{bmatrix} L_3 \\ -L_3 \\ L_3 \end{bmatrix}^T$$

$$= \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{1}{3} \end{bmatrix} + \begin{bmatrix} \alpha L_3 & -\alpha L_3 & \alpha L_3 \\ \beta L_3 & -\beta L_3 & \beta L_3 \\ \gamma L_3 & -\gamma L_3 & \gamma L_3 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{3} + \alpha L_3 & \frac{1}{3} - \alpha L_3 & \frac{2}{3} + \alpha L_3 \\ \frac{1}{3} + \beta L_3 & \frac{2}{3} - \beta L_3 & \frac{1}{3} + \beta L_3 \\ \frac{2}{3} + \gamma L_3 & \frac{1}{3} - \gamma L_3 & -\frac{1}{3} + \gamma L_3 \end{bmatrix}$$

Now for

$$L_3 = 1, \alpha = \frac{1}{3}, \beta = -\frac{1}{3}, \gamma = \frac{1}{3}$$

we have

$$Z = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Next, given Z for any vector of three agents in input we can generate another evaluation as output. For example, for given

$$X(p) = \begin{pmatrix} g_1 & g_2 & g_3 \\ 1 & 1 & 0 \end{pmatrix}$$

and Z we get

$$Y(p) = ZX(p) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

B. Loops and logic projection operators

Now we will explore a loop in Fig. 5 for the AUT Logic network.

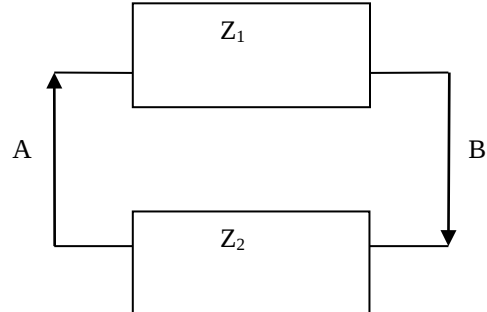


Fig. 5. Loop in the AUT Logic network

The equation (8) gives us

$$Z_1 = B (A^T A)^{-1} A^T, \quad Z_2 = A (B^T B)^{-1} B^T$$

and

$$Q1 = Z_1 Z_2 = B (A^T A)^{-1} A^T A (B^T B)^{-1} B^T = B (B^T B)^{-1} B^T$$

Knowing $Q1 B = B$, we call $Q1$ a *projection operator*. It has a property

$$Q1^2 = B (B^T B)^{-1} B^T B (B^T B)^{-1} B^T = B (B^T B)^{-1} B^T = Q1$$

In the same loop we have another projection operator $Q2$:

$$Q2 = Z_2 Z_1 = A (B^T B)^{-1} B^T B (A^T A)^{-1} A^T = A (A^T A)^{-1} A^T$$

with properties $Q2 A = A$, and

$$Q2^2 = A (A^T A)^{-1} A^T A (A^T A)^{-1} A^T = A (A^T A)^{-1} A^T = Q2$$

C. Geometric image of the logic projection operator

Next we show the geometric image of the projection operator $Q2$ for the matrix in a simple case of:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2} \end{bmatrix} = A_1 \oplus A_2 = \begin{bmatrix} a_{1,1} \\ a_{2,1} \\ a_{3,1} \end{bmatrix} \oplus \begin{bmatrix} a_{1,2} \\ a_{2,2} \\ a_{3,2} \end{bmatrix}$$

and

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

Here

$$Q2X = A(A^T A)^{-1} A^T X = AS = S_1 \begin{bmatrix} A_{1,1} \\ A_{2,1} \\ A_{3,1} \end{bmatrix} + S_2 \begin{bmatrix} A_{1,2} \\ A_{2,2} \\ A_{3,2} \end{bmatrix} = S_1 A_1 + S_2 A_2$$

and the geometric image of the projection operator is shown in Fig. 6 that shows the projection operator $Q2$ as a loop in the AUT logic network from Fig.5.

In Fig. 6, X is a 3-D vector which axes are P_1, P_2, P_3 , that represent one dimension for any agent. The operator $Q2$ projects the vector X into the space A_1, A_2 of the columns of A . S_1 and S_2 are the components of $Q2X$ in the space of A_1 and A_2 .

V. CONCLUSION

This paper had shown that it is possible to create an AUT logic network that produces matrix logic operations from given logic evaluation samples for a set of agents.

These operations serve as changers of original agents' evaluations.

In this way, we suggest a new type of logic computation for uncertainty situation and a new software logic control approach. The new network extends the classical Boolean

network, where the states are logic propositions that assume 1 and 0 values. In our case, the variables are Boolean vectors where components of the vectors are the logic evaluations given by different agents.

The proposed the AUT network theory generalizes the uncertainty theories such as evidence theory, probability theory, and fuzzy sets because all of them model agents that are rational relative to tautology ($p \vee \neg p$) that is always true and contradiction ($p \wedge \neg p$) that is always false. The AUT allows reasoning under uncertainty for the sets of agents that combine rational and irrational agents.

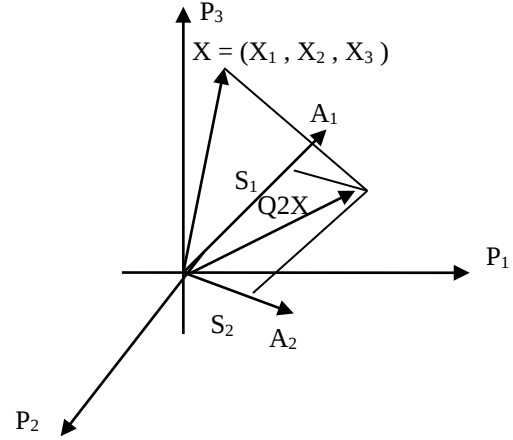


Fig. 6. Projection operator $Q2$ as a loop in the AUT logic network

The goal of this paper was to lay down the theoretical foundation and to formalize intuitive ideas.

The next step of this research is to work on outlining the applications areas where this approach will be beneficial and conducting computational simulation experiments to compare the proposed technique with the other existing techniques in the current fuzzy logic applications.

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